The Blues and the Abstract Truth: Music and Mental Models*

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Draft, November 5, 1999

1 Introduction

The idea that there is a grammar of music is probably as old as the idea of grammar itself, and the idea that there should be *formal* grammars of music followed equally hard upon the Chomskean application to natural languages of the formal techniques used to analyse logical and mathematical languages – see Winograd 1968, Lindblom and Sundberg 1972, Steedman 1973, 1977, Longuet-Higgins 1978, Johnson-Laird 1991, and many others reviewed by Sundberg and Lindblom 1991.

Nevertheless, it is probably fair to say that such musical formal grammars have lagged behind the linguistic ones in terms of descriptive adequacy, constraint on formal generative power, psychological plausibility, and computational practicality, especially as far as harmonic analysis goes.

For example, Steedman 1984 offers a generative grammar for chord progressions in jazz twelve-bar blues. The grammar is used to account for the fact that the chord sequences shown in Figure 1 – the last of which shares no bars in common with the first, other than the fifth and final bars – are perceived by jazz musicans as being in some sense "paraphrases". That is, they are all instances of the twelve bar form, one of the basic forms of the music. (The notation, which is for the most part standard, is explained in an appendix to the present chapter.)

The 1984 paper attempts to capture this fact in a small number of rules modeled on the linguist's device of a "rewriting rule". The principal rules are shown in Figure 2, in which X is a variable over relative chord roots I, II, etc, and IV_X is to be read as "the chord that is X's IV (so that if X is II, IV_X is V).[1] I will briefly summarise the motivation for these rules, but the reader is directed to the earlier paper for a fuller account.

Rule 0 simply defines the skeleton of the twelve bar as two two bar units of tonic I harmony, followed by two two bar units constituting a "plagal cadence" – a progression

*To appear in: A. Garnham and J. Oakhill (eds.)*Mental Models in Cognitive Science*, Erlbaum, Mahwah, NJ, 1996

$a)$	$I(M7)$	$IV(7')$	$I(M7)$	$I7$	$IV(7')$	$IV(7')$	$I(M7)$	$I(M7)$	$V7$	$V7$	$I(M7)$	$I(M7)$
$b)$	$I(M7)$	$IV(7')$	$I(M7)$	$Vm7, I7$	$IV(7')$	$\sharp IV \circ 7$	$I(M7)$	$VI7$	$IIm7$	$V7$	$I(M7)$	$I(M7)$
$c)$	$I(M7)$	$IV(7')$	$I(M7)$	$Vm7, I7$	$IV(M7)$	$IVm7$	$IIIm7$	$VI7$	$IIm7$	$V7$	$I(M7)$	$I(M7)$
$d)$	$I(M7)$	$IIm(7'), \sharp II \circ 7$	$IIIm(7')$	$Vm7, I7$	$IV(M7)$	$IVm7, \flat VII7$	$IIIm7$	$\flat IIIm7$	$IIm7$	$V7$	$I(M7)$	$I(M7)$
$e)$	$I(M7)$	$VII\phi 7, III7$	$VIm7, II7$	$Vm7, I7$	$IV(M7)$	$IVm7, \flat VII7$	$\flat III(M7)$	$\flat IIIm7, \flat VI(7')$	$IIm7$	$V7$	$I(M7)$	$I(M7)$
$f)$	$I(M7)$	$IV(7')$	$I(M7)$	$\flat IIm7, \flat V7$	$IV(7')$	$\sharp IV \circ 7$	$IIIm7$	$VI7$	$IIm7, V7$	$\flat VIm7, \flat II7$	$I(M7)$	$I(M7)$
$g)$	$\flat II7, \flat V7$	$VII7, III7$	$VI7, II7$	$V7, I7$	$IV(7')$	$\sharp IV \circ 7$	$IIIm7$	$\flat III7$	$VIm7$	$\flat II7$	$I(M7)$	$I(M7)$

Figure 1: Some Jazz 12-bars (adapted from Coker, 1964)

0.			$12bar$	$\rightarrow$	$I \quad I7$	$IV \quad I$	$V7 \quad I$
1.			$X(m)(7)$	$\rightarrow$	$X(m)$	$X(m)(7)$	
2.			$X(m)$	$\rightarrow$	$X(m)$	IV_X	
$3a.$		W	$X7$	$\rightarrow$	$V_X(m)7$	$X7$	
$3b.$		W	$Xm7$	$\rightarrow$	$V_X 7$	$xm7$	
4.		$V_X 7$	$X(m)(7)$	$\rightarrow$	$\flat II_X(m)(7)$	$X(m)(7)$	
5.	X	X	X	$\rightarrow$	X	$II_X m$	$III_X m$
$6a.$	$X(m)$	$X(m)$	V_X	$\rightarrow$	$X(m)$	$\sharp X \circ 7$	V_X
$6b.$	$X(m)$	$X(m)$	$II_X m7$	$\rightarrow$	$X(m)$	$\sharp X \circ 7$	$II_X m7$
$6c.$	$X(m)$	$X(m)$	$VII_X m7$	$\rightarrow$	$X(m)$	$\sharp X \circ 7$	$VII_X m7$

Figure 2: Chord Substitution Rules for Jazz 12-bars (adapted from Steedman, 1984)

from IV to I – followed by two two bar units constituting a "perfect cadence", from V to I.

Rule 1 recursively expands this framework as a binary tree. There is a convention not indicated in the rules whereby the total number of bars or fractions thereof on the right equals that on the left – hence each of the Xs on the right of rule two lasts half as long as the one on the left. The brackets round the minor annotations m mean that the relevant chords can optionally be minor, and there is another convention that says that if the thing on the left is minor the things on the right must be. Bracketed 7 means that those chords are optionally dominant sevenths and a similar inheritance convention applies. Note that the dominant seventh when present passes down the *right branch*. This is important because a dominant seventh strongly constrains the chord that follows.

Rule 2 is another binary tree expansion rule, with the same conventions, which says that a chord X *other than* a dominant seventh can be expanded as that chord and its subdominant IV_X. Rules 0 to 2 are all that is required to generate the chord roots for the mind-numbingly dull twelve bar (a) in Figure 1.

The remainder of the rules are rules which substitute more interesting chords in this somewhat boring framework, to achieve the variety illustrated in the remainder of Figure 1. Rule 3, which comes in two instances, does most of the interesting work in this respect. 3a says that any chord W preciding a major dominant seventh chord $X7$ can be replaced by the dominant seventh of X, $V_X 7$ or the minor dominant seventh of X, $V_X m7$. 3b says the same applies to the chord preciding a *minor* dominant seventh on X, except that the

$7a.\quad X \quad \rightarrow \quad \{X(M7), X(7'), X9, X13\}$
$7b.\quad X7 \quad \rightarrow \quad \{X\flat 9, X\flat 10, X7+5\}$
$7c.\quad Xm \quad \rightarrow \quad \{Xm(7'), Xm6\}$
$7d.\quad Xm7 \quad \rightarrow \quad \{Xm9, X\phi 7\}$

Figure 3: Surface "Spellings" of Chords (adapted from Steedman, 1984)

new chord has to be major. The chord W in these rules is further limited to chords that have not been affected by any previous substitution by a root-changing rule like 3 itself (see Steedman 1984, p.63.

Rules 4, 5, and 6 introduce various passing chords whose detailed motivation need not detain us here.

Together with the trivial rules shown in Figure 3 for optionally adding further notes to the basic chord types X, $X7$, Xm, $Xm7$ the earlier paper shows that the grammar covers a small corpus of Jazz twelve-bars including the above representative examples. (Again the conventional interpretations of these chord symbols are given in the appendix.)

There are a number of things that are unsatisfactory about this grammar. One is its minute coverage. However, the work of Johnson Laird 1991 shows that rather similar kinds of rules can be generalised to a more diverse set of harmonic "skeletons" common in jazz, including the very frequent "I've Got Rhythm" family.

Another objection that has been raised is that the rules give the appearance of Context-sensitive PS rules, a very powerful class of grammars indeed. I shall show below that this appearance is actually illusory. We can replace these rules (and the corresponding rules in Johnson-Laird – see p.311) by strongly equivalent context-free rules.

The third objection is that, while rules 2 through 7 capture the musician's intuition that elaborated chord sequences are derived from simpler ones by a process of chord substitution, the way the rules are phrased, and in particular the presence of the variable W, even with the rather nasty condition that W not have been altered by any previous substitution, means that the search space for the parser is large. This difficulty can to some extent be overcome by suitable search strategies. However, these have proved hard to identify without compromising the integrity of the grammar itself (see Mouton and Pachet 1995). Moreover, the most promising search strategies merely point to a further implausibility in the grammar, as follows.

Rule 3 has the effect of propagating perfect cadences backwards. That is, successive substitutions in the basic skeleton (a) of Figure 1 generate examples like those in Figure 4, in which the elaborated cadence is underlined: This means that the value of, for example, the $IIIm7$ chord in a''' in Figure 4 is dependent upon a chain of substitutions working back from a quite distant $V7$ to its right. This is suggests that a good parsing strategy to minimise search is to parse from right to left. (The same strategy was also a forced move for the parser proposed by Winograd 1968.

Musically it is quite correct to claim that the $IIIm7$ is dependent upon the $V7$ to its

$a.$	$I(M7)$	$IV(7')$	$I(M7)$	$I7$	$IV(7')$	$IV(7')$	$I(M7)$	$I(M7)$	$V7$	$V7$	$I(M7)$	$I(M7)$
$a'.$	$I(M7)$	$IV(7')$	$I(M7)$	$I7$	$IV(7')$	$IV(7')$	$I(M7)$	$I(M7)$	$IIm7$	$V7$	$I(M7)$	$I(M7)$
$a''.$	$I(M7)$	$IV(7')$	$I(M7)$	$I7$	$IV(7')$	$IV(7')$	$I(M7)$	$VI7$	$IIm7$	$V7$	$I(M7)$	$I(M7)$
$a'''.$	$I(M7)$	$IV(7')$	$I(M7)$	$I7$	$IV(7')$	$IV(7')$	$IIIm7$	$VI7$	$IIm7$	$V7$	$I(M7)$	$I(M7)$

. . . . $etc.$ . . . .

Figure 4: Effect of Recursive application of Rule 3

right, and since this grammar is in Chomsky's terms a "competence grammar", distinct from the performance mechanism that delivers the analysis, there is nothing in principle wrong with this assumption about the parser. Nevertheless, it is in psychological terms quite surprising to be find that the optimum processor for our grammar is right to left. Our own experience of such music does not suggest that we need to wait to the end of a chord sequence like a''' to interpret the role of that $IIIm7$ chord. As in the case of natural language processing, our intuition is that we interpret sequences more or less note by note and chord by chord, from left to right. In the case of natural language processing, at least, there is abundant experimental evidence to back this intuition up (see Marslen-Wilson 1975, Tyler and Marslen-Wilson 1977, and much subsequent work.)

2 Longuet-Higgins' Theory of Tonal Harmony

To devise a better grammar, we need to get away from the whole idea of substituting one chord for another, and to seek something founded more straightforwardly in musical semantics.

The first completely formal identification of the nature of the harmonic relation is in Longuet-Higgins (1962a, 1962b), although there are some earlier incomplete proposals, including work by Weber, Schoenberg, Hindemith, and the important work of Ellis (1874, 1875). Longuet-Higgins showed that the set of musical intervals relative to some fundamental frequency was the set of ratios definable as the product of powers of the prime factors 2, 3, and 5, and no others – that is as a ratio of the form $2^x.3^y.5^z$, where x, y, and z are positive or negative integers. (The fact that ratios involving factors of seven and higher primes do not contribute to this definition of harmony does not exclude them from the theory of consonance. In real resonators, overtones involving such factors do arise, and contribute to consonance. Helmholtz realised that the absence of such ratios from the chord system of tonal harmony represented a problem for his theory of chord function, and attempted an explanation in terms of consonance – see Ellis (translation) 1885, p.213).[2]

Longuet-Higgins' observation means that the intervals form a three-dimensional discrete space, with those factors as its generators, in which the musical intervals can be viewed as vectors. Since the ratio 2 corresponds to the musical octave, and since for most harmonic purposes, notes an octave apart are functionally equivalent, and have the same note-names, it is convenient to project the three dimensional space along this axis into the 3 x 5 plane, assigning each position its traditional note-name. It is convenient to plot the plane relative

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Figure 5: (Part of) The Space of Note-names (adapted from Longuet-Higgins 1962a)

to a central C, when it appears as in Figure 5, adapted from Longuet-Higgins (1962a).

The traditional note names are ambiguous with respect to the intervals, and the pattern of names repeats itself in a south-easterly direction, although each position necessarily represents a unique frequency ratio when played in just intonatation. (That is to say that the note names "wrap" the plane of musically significant frequency ratios onto a cylinder, which is here projected back onto the plane). Nevertheless, every vector in the infinite plane from some origin necessarily corresponds to a distinct frequency ratio, and potentially to a distinct musical function. There is a traditional nomenclature which distinguishes among the different functions corresponding for example to the two Ds relative to the central C in figure 5, as between the "major tone" and the "minor tone". However, this nomenclature is confusing and not entirely systematic. Instead we will display the intervals relative to an origin or tonic I using the same roman numeral notation as is used for the chord roots in Figure 1, as in Figure 6. In this figure the intervals are disambiguated. The prefix $\sharp$ and $\flat$ roughly correspond respectively to the traditional notions of "augmented" intervals, and to "minor" and/or "diminished" intervals, while the superscripts plus and minus roughly correspond to the "imperfect" intervals. (However the intervals here identified as II^{-}, $\flat VII^{-}$, and $\flat V^{-}$ would usually be referred to as the minor tone, dominant seventh, and minor fifth, rather than as imperfect intervals, and the interval shown as $\sharp IV$ should be referred to as the tritone, rather than the augmented fourth). The positions with no prefixes and suffixes are "major" and/or "perfect" intervals.

Crucially for our purpose, if we choose a particular position X in the plane of notenames of Figure 5 as origin, and then superimpose the plane of intervals in roman numeral notation of Figure 6, with the I over the X, then we can calculate note names corresponding to intervals like II_X, VII.[3]

III$^-$	VII$^-$	#IV$^-$	#I	#V	#II	#VI	#III$^+$	#VII$^+$
I$^-$	V$^-$	II$^-$	VI	III	VII	#IV	#I$^+$	#V$^+$
bVI$^-$	bIII$^-$	bVII$^-$	IV	I	V	II	VI$^+$	III$^+$
bIV$^-$	bI$^-$	bV$^-$	bII	bVI	bIII	bVII	IV$^+$	I$^+$
bbII$^-$	bbVI$^-$	bbIII$^-$	bbVII	bIV	bI	bV	bII$^+$	bVI$^+$

Figure 6: (Part of) The Space of Disambiguated Harmonic Intervals

Longuet-Higgins' harmonic representation therefore bears a strong resemblance to a "mental model" in the sense of Johnson-Laird 1983. That is to say that it builds directly into the representation some of the properties of the system that it represents. It will be obvious to musicians that the intervals that they refer to as harmonically remote, such as the imperfect and augmented intervals, are spatially distant from the origin in the representation. Similarly, the definition of musically coherent chord sequences such as the twelve-bar blues has something to with orderly progression to a destination by small steps in this space.

For example, the basic sequence in Figure 1a, repeated as Figure 4a, is a closed journey around a central I visiting the immediately neighbouring IV and V. Figure 4d makes a jump to the right to II, then returns via V. Figure 4d'' is perhaps the most interesting, because it takes a step up to III, then proceeds via leftward steps to end up on I^-. (This is a progression used to great effect by Louis Armstrong (1927) on *Basin Street Blues*, although its original discovery is at least as early as Beethoven's G major Piano Concerto, as Longuet-Higgins has pointed out.)

I^- is musically distinct from the original I, and if perfectly intoned (as opposed to being played on an equally tempered keyboard), would differ from the original in a ratio of 80:81. Nevertheless, we are able to treat it as the tonic.

This theory also explains why the dominant seventh chord creates such a strong expectation of a following chord to its left, whereas the same chord without does not. The major chord on a root V, shown in Figure 7 as made up of a circled V, VII, and II, is extremely unambiguous as to its interpretation, like all such triads. Thus, even if the major triad is played on an equally tempered instrument, obscuring the distinction between the frequency ratios of the pure intervals, having picked *that* V, the representation makes it obvious why the harmonically closest interpretations of the VII and the II are not any of the imperfect

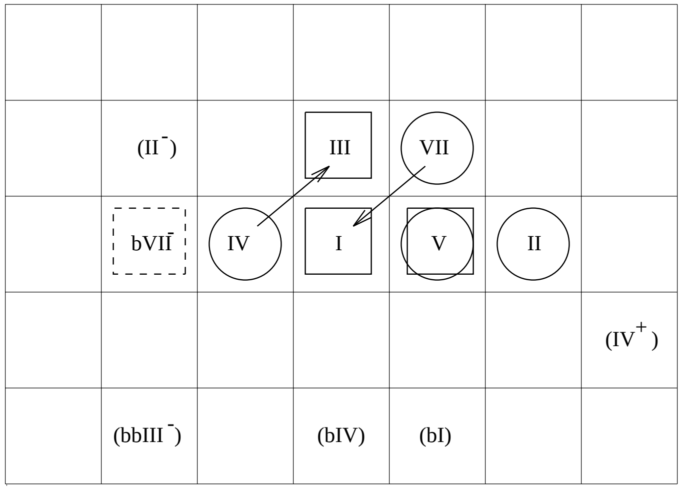


Figure 7: The Interpretation of the Dominant Seventh Chord (circles) and its resolution (squares)

or diminished alternatives shown in brackets. However, it is the addition of the dominant seventh of V, the circled IV, that makes the V chord have a hole in its middle, into which a triad on I (squared I, III, V) fits neatly, sharing one note with the first chord, and with the two remaining notes standing in semitone "leading note" relations with two other notes in the first chord.[4] A chord of I is indeed the expectation produced by a dominant seventh chord $V7$. Moreover the addition of a dominant seventh $\flat VII^{-}$ to the I major triad (dotted square) makes the I in turn lead onto the IV to its left. The effect of adding dominant seventh chords to *minor* triads is suggested as an exercise at this point. (Why is an alternation of major and minor dominant seventh chords so effective?)

This "semantics" for the dominant seventh chord, showing why it generates a leftward shift in the space and a "need" or expectation for the corresponding tonic, is crucial to explaining why the grammar developed in the next section takes the form that it does. This representation or mental model is what underlay the earlier grammar, as I pointed out in passing at the time (Steedman, 1984, p.56, n.4). However, to make explicit the way in which this works points the way to a different "type-driven" kind of grammar, which makes explicit in the category of a dominant seventh chord its need for the tonic. Such a grammar will point the way to removing the distressingly right-branching analyses of the earlier version.

3 Categorial Grammar

Categorial Grammars for natural languages have been mainly advanced as competence grammars, for linguistic reasons (see Wood 1993 for a review). However, one branch of the family, the flexible or "combinatory" categorial grammars, which include associative operations like function composition, have also been defended on the grounds that they allow left-branching analyses of what are traditionally viewed as right-branching constructions, and are therefore more directly compatible with processors that make available incrementally assembled semantic analyses at an early stage in processing (see Steedman 1989 for the argument).

Like other lexicalist approaches, categorial grammars put into the lexicon most of the information that is standardly captured in context-free phrase-structure rules. For example, instead of using rules like 1 to capture the basic syntactic facts concerning English transitive sentences, such grammars associate with English transitive verbs a category which we will usually write as in 2:[5]

(1)
$$\begin{array}{lll} S & \rightarrow & NP\ \ VP \\ VP & \rightarrow & TV\ \ NP \\ TV & \rightarrow & \{\mathrm{eats}, \mathrm{drinks}, \ldots\} \end{array}$$

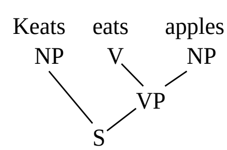
(2) eats := $(S\backslash NP)/NP$

The category says that *eats* is a *function*, combining with an NP to its right to yield a predicate, which is itself a function bearing the category $S\backslash NP$, which in turn combines with an NP to its left to yield an S.[6] Combination takes place via the following rules of functional application, which in a pure categorial grammar are the only rules of combination:

(3) *Functional Application:*
$$\begin{array}{llll} a. & X/Y \quad Y & \Rightarrow & X \\ b. & Y \quad X\backslash Y & \Rightarrow & X \end{array}$$

These rules have the form of very general binary PS rule schemata. In fact Categorial Grammar is just binary-branching context-free grammar written in the accepting, rather than the producing, direction. There is a consequent transfer of the major burden of specifying particular grammars from the PS rules to the lexicon. While it is now convenient to write derivations as in a, below, they are equivalent to conventional trees, as in b.

(4) a.
Keats eats apples
$\overline{NP}$ $\overline{(S\backslash NP)/NP}$ $\overline{NP}$
$\overline{\qquad\qquad S\backslash NP \qquad\qquad}^{>}$
$\overline{\qquad\qquad\qquad S \qquad\qquad\qquad}^{>}$

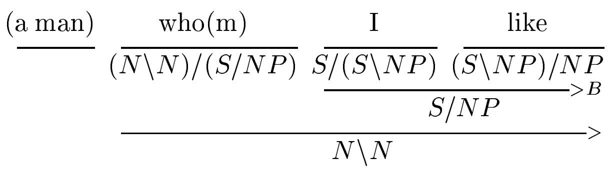
b.


(The annotations $>$ and $<$ on combinations in a, above, are mnemonic for the rightward and leftward function application rules 3a,b).

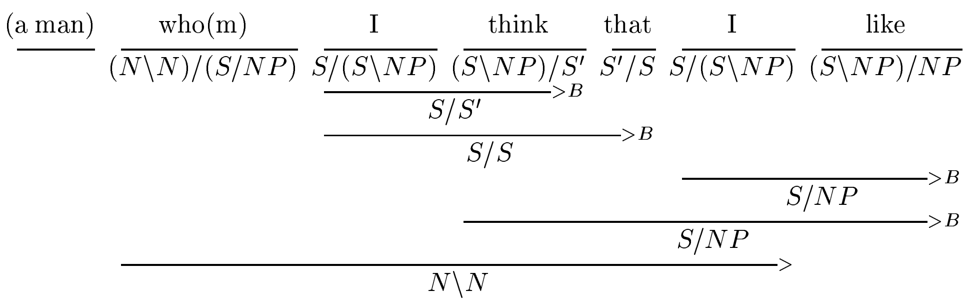
Flexible categorial grammars typically handle relative clauses by allowing rules of functional composition, such as the following:

(5) *Forward Composition* (>B):
$$X/Y \quad Y/Z \quad \Rightarrow_{\mathbf{B}} \quad X/Z$$

Together with "type-raised" categories that can be substituted in the lexicon or introduced by rule for argument categories like subject NPs, rule 5 allows extractions as follows

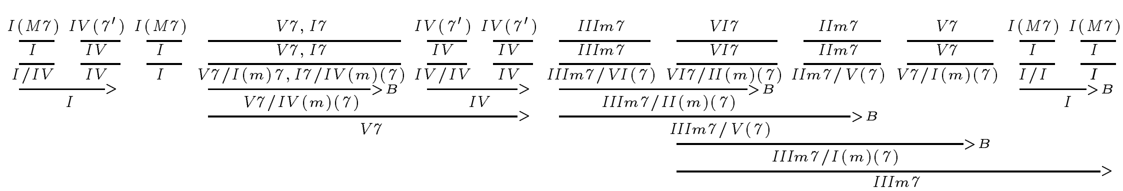
(6)


Such extractions are immediately predicted to be unbounded:

(7)


The interesting thing about such grammars for present purposes is that they allow left-branching analysis of structures like the english clause, which we usually think of as predominantly right-branching. While we will not go into the details here, it is also crucial that the combinatory rules allow the immediate assembly of a correct semantic interpretation for non-standard constituents like *I think that I like*. The implications of this fact for the theory of natural language grammar are quite far-reaching, and are explored in Steedman 1995.

Such grammars afford an equally natural expression for the harmonic semantics of the last section. Figure 8 gives a categorial lexicon that corresponds point for point with the earlier grammar in Figure 2. The categories are numbered accordingly to facilitate the comparison (conventions for inheriting optional properties like (m) are as before). Together with the same "spelling" rules as before (Figure 3), and function composition and type-raising as well as function application, this grammar gives rise to (incomplete) derivations like the following for the chord sequence *c* in Figure 1:

(8)


$$
\begin{array}{llcc}
(1). & X(m) & := & X(m)(7)/X(m)(7) \\
 & & & X(m) \\
(2). & X(m) & := & X(m)/IV_X \\
(3a.) & Xm7 & := & Xm7/IV_X(7) \\
(3b.) & X7 & := & X7/IV_X(m)(7) \\
(4.) & X(m)7 & := & \sharp IV_X(m)7/VII_X(m)(7) \\
(5.) & X & := & (X/III_X m)/II_X m \\
(6.) & X\circ 7 & := & \flat V_X/\flat V_X \\
 & & & \flat II_X/\flat II_X \\
 & & & \flat VII_X m7/\flat VII_X m7
\end{array}
$$

Figure 8: A Categorial Grammar equivalent to Figure 2

There are a number of things to notice about this fragment. First, unlike its predecessor, it does not work by substitution on a previously prepared skeleton. Secondly, it is still incomplete, in that it does not yet specify the higher levels of analysis that stitch the sequences of cadences together into canonical forms like twelve-bars, and variations on *I Got Rhythm*. (Notice that it as yet includes no lexical categories corresponding to Rule 0 in the old grammar of Figure 2). However, the lineaments of Johnson-Laird's more elaborated 1991 cadence grammar are visible to the willing eye in the categories of Figure 8.

We can show this by including some further unary type-changing rules, analogous to the type-raising in of categories like subjects in natural language CG mentioned in connection with example 6. (These rules actually smuggle the equivalent of Rule 0 or Johnson-Laird's PS rules back into the grammar).

First, instead of just applying an extended cadence to its target, we will give the target a higher-order type that labels the result explicitly as C, the category of a non-initial cadence:

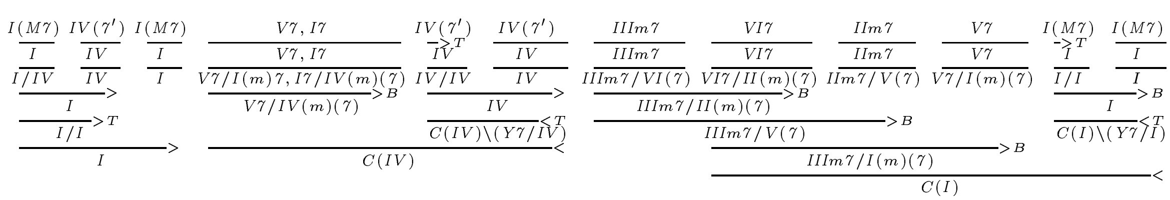
(9) $\quad X \;\Rightarrow\; C(X)\backslash(Y7/X)$

Second, we add a trivial rule that makes X into X/X – that is, which allows strings of Xs to combine into one X (this rule renders redundant the categories 1 in figure 8).

(10) $\quad X \;\Rightarrow\; X/X$

With these rules we can continue the earlier derivation as follows:

(11)



The analysis is not yet complete, but we are at least in a position to distinguish twelve-bar sequences made up from cadences onto I, IV and V from other less coherent sequences. (We assume as before that the combination of two categories X and Y each respectively occupying x and y bars creates an object occupying $x + y$ bars).

While more work remains to be done before this fragment will support analysis or generation, this is a much nicer *kind* of grammar than the one I offered in 1984. The fact that the only rules are "order-preserving" presumably means that it is weakly context-free (although more needs to be said about the type-changing rules). And although I have not provided anything like a formal semantics or model theory, the fact that this is a one-level grammar, rather than one based on substitutions, suggests that such a semantics would be rather easily specifiable in terms of the Longuet-Higgins theory, as the resemblance to Montague Grammar should suggest. It also looks as if this grammar will be fairly simply parsable, and incrementally interpretable from left-to-right. To that extent it may constitute an advance on its predecessor, and perhaps lead to similar simplifications in the more ambitious but similarly right-branching grammars of Winograd and Johnson-Laird.

Appendix: Conventions Used in Figure 1

The sequences (a) to (g) represent the 12-bar chord sequences. Vertical columns represent the 12 successive bars, further grouped into four-bar sections. Where only one chord symbol occurs in a bar it is to be understood to last for all four beats of the bar. Where there are two symbols, they each occupy two beats. The root of each chord is identified by a Roman numeral. This indicates a degree in the major scale of the keynote of the piece, I being the tonic and VII the seventh. The prefixes $\flat$ and $\sharp$ identify the root of the chord in question as being one semitone above or below the degree in question. For example, $\flat III$ indicates a chord whose root is the minor third of I. All chords are understood to be based on the major chord unless explicit indication is given that they are based on the minor by a small m immediately following the Roman numeral, as in $\flat IIIm$. Further numerical suffixes indicate that additional "passing" notes are to be included with the notes of the basic minor or major chord. The ones in brackets are less harmonically significant, in the sense spelled out earlier in the discussion of the original rules in Figure 3. Their identity is indicated in a rather obscure (but standard) way. The suffix 7 means that the "dominant" seventh note, a tone below the tonic, is to be included, as in $\flat III7$ and $IIIm7$. The nonstandard suffix (7') also denotes a keyboard tone below the tonic. However, in these chords the additional note functions as the *minor* seventh, rather than the dominant seventh – cf. footnote 4.) The suffix (M7), in contrast, indicates the inclusion of the leading note or major seventh, a semitone below the root, as in $IV(M7)$. The suffix $+5$ indicates the addition of the note an augmented fifth above the tonic (G$\sharp$ for the chord of C). It often occurs in combination with the dominant seventh, as in $V7 + 5$.

The suffix 6 indicates that the major sixth is added. The suffix ϕ7 indicates that the minor third, the diminished fifth (G$\flat$ for the chord of Cϕ7), and the dominant seventh are included. The suffix $\circ$7 indicates that the minor third, the diminished fifth, and the diminished seventh

(B♭♭ for the chord of C∘7) are all included – this is the so-called diminished seventh chord.

Acknowledgments

The research described above has its origins in many hours of work with Phil Johnson-Laird, Keith Oatley, and Mark Georgeson in the Epistemic Spasm Band at the University of Sussex in the mid-70s, while the author was a postdoctoral fellow in Phil's Lab under SSRC grant HR 2351/1. (They don't write numbers like that any more). The author hereby records his lasting gratitude for this appointment, which allowed him to switch fields into Natural Language Processing, and from which he learned much of what he knows about the subject.

More recent phases of the research have been supported in part by NSF grant nos. IRI91-17110, IRI95-04372, ARPA grant no. N66001-94-C6043, and ARO grant no. DAAH04-94-G0426. Thanks to Brian Butterworth and Oliver Nelson for suggesting the first part of the title, and to Alan Garnham for a close reading of the draft.

Notes

[1]This notation is different from, but equivalent to, that in the original paper.

[2]The history of these developments and some related developments in work of Balzano 1982, Shepard 1982 and Lerdahl 1988 is reviewed in greater detail by Steedman 1994.

[3]A simple analogue calculator for this purpose can readily be built by photocopying the roman numeral interval plane of Figure 6 onto transparent film, and then sliding it over the note-name plane, Figure 5.

[4]The addition of the new note also makes the V chord rather ambiguous. The added IV could be the south-easterly IV^{+}, making this a minor seventh $V(7')$ chord rather than a dominant seventh $V7$.

[5]This is the "result leftmost" notation for categories. There is another "result on top" notation in use, in which this category would be written $(NP\backslash S)/NP$.

[6]Of course, example 2 is not the *only* category that the verb *eats* bears. Like many other transitives, it can also be used intransitively, as $S\backslash NP$, like *walks*. For parsing purposes, we might combine such categories into a single disjunctive lexical entry, including optional arguments. However, such considerations are irrelevant to competence grammar, and we shall here treat such alternatives as independent lexical categories.

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