Formal Grammars for Computational Musical Analysis:

The Blues and the Abstract Truth

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Supposing, for instance, that the fundamental relations of pitched sounds in the science of harmony and of musical composition were susceptible of such expression and adaptations, the Engine might compose elaborate and scientific pieces of music of any degree of complexity and extent.
Ada Lovelace (1842), “Translator’s notes to an article on Babbage’s Analytical Engine”

The Jazz Blues as a Language

- There are in principal infinitely many variations on the primordial Twelve-bar Blues chord sequence that jazz musicians recognize as such, just as we all understand the infinite variety of English sentences.
- They have been explored by the likes of Louis Armstrong, Charlie Parker, and our contemporaries.

a)	I(M7)	IV(7)	I(M7)	I(7)	IV(7)	IV(7)	I(M7)	I(M7)	V(7)	V(7)	I(M7)	I(M7)
b)	I(M7)	IV(7)	I(M7)	Vm(7), I(7)	IV(7)	♯IV∘7	I(M7)	VI(7)	IIm(7)	V(7)	I(M7)	I(M7)
c)	I(M7)	IV(7)	I(M7)	Vm(7), I(7)	IV(M7)	IVm(7)	IIIm(7)	VI(7)	IIm(7)	V(7)	I(M7)	I(M7)
d)	I(M7)	IIm(7), ♯II∘7	IIIm(7)	Vm(7), I(7)	IV(M7)	IVm(7), ♭VII(7)	IIIm(7)	♭IIIm(7)	IIm(7)	V(7)	I(M7)	I(M7)
e)	I(M7)	VII(ϕ7), III(7)	VIm(7), II(7)	Vm(7), I(7)	IV(M7)	IVm(7), ♭VII(7)	♭III(M7)	♭IIIm(7), ♭VI(7)	IIm(7)	V(7)	I(M7)	I(M7)
f)	I(M7)	IV(7)	I(M7)	♭IIm(7), ♭V(7)	IV(7)	♯IV∘7	IIIm(7)	VI(7)	IIm(7), V(7)	♭VIm(7), ♭II(7)	I(M7)	I(M7)
g)	♭II(7), ♭V(7)	VII(7), III(7)	VI(7), II(7)	V(7), I(7)	IV(7)	♯IV∘7	IIIm(7)	♭III(7)	VIm(7)	♭II(7)	I(M7)	I(M7)

Figure 1: Some Jazz 12-bars (adapted from Coker, 1964)

"Phonological Spelling" of Chords

- We can regard all of the chords in figure 1 as falling into four basic chord types within which they differ only as to which particular additional notes they add. The four types are distinguished as to whether they are the major chord X or the minor chord Xm, and whether they include the dominant seventh note, when they are written X^7 and Xm^7 respectively

(1) *a*. $\{X(M7), X(7), X(9), X(13)\} := X$

b. $\{Xm(7), Xm(6)\} := Xm$

c. $\{X(7), X(\flat 9), X(\flat 10), X(7+5)\} := X^7$

d. $\{Xm(7), Xm(9), X(\phi 7)\} := Xm^7$

- The "dominant seventh" chords X^7 and Xm^7 create an expectation of IV_X the chord of the 4th degree of the scale relative to X as I or tonic. X and Xm do not create a particular expectation of this kind.

Phonological Spelling of Chords (Contd.)

- A dominant seventh chord followed by that expected IV chord is an elementary perfect or authentic cadence.
- The phonological spelling level of rules is is where issues of voice-leading and inversion should be brought into the grammar, analogously to processes like liaison and lenition in speech.
- It is also where issues of ambiguity come in—X and X^7 can both be realized as $\mathrm{X}(7)$, since the minor seventh and dominant seventh are homophones in equal temperament.

The Recursive Nature of Cadences

- You can derive more complicated blues chord sequences from simpler ones by propagating authentic cadences backwards. That is, successive substitutions in the basic skeleton (a) of Figure 1 generate examples like those in Figure 2, in which the elaborated cadence is underlined:

$a.$	I	IV	I	I^7	IV	IV	I	I	V^7	V^7	I	I
$a'.$	I	IV	I	I^7	IV	IV	I	I	IIm^7	V^7	I	I
$a''.$	I	IV	I	I^7	IV	IV	I	VI^7	IIm^7	V^7	I	I
$a'''.$	I	IV	I	I^7	IV	IV	$IIIm^7$	VI^7	IIm^7	V^7	I	I

. . . . *etc.* . . . .

Figure 2: Recursive Propagation of the Authentic Cadence

Prolonged Cadences as Right-branching Trees

- The value of, for example, the IIIm^7 chord in a''' in Figure 2 is therefore dependent upon a chain of substitutions working back from a quite distant V^7 to its right, suggesting a *right-branching* tree-structure characteristic of many Schenkerian approaches (including Steedman 1984).

- This suggests (paradoxically) that a good parsing strategy to minimize search is to parse from right to left.

- For example, this was the strategy used in the Systemic Grammar-based musical parser proposed by Winograd 1968.

- But this is an anomaly. We experience both musical meaning and linguistic meaning incrementally from the beginning.

- Even if we recognize a musical "competence performance distinction," the anomaly persists. (Why *doesn't* performance reflect competence?)

Longuet-Higgins' Theory of Tonal Harmony

- To devise more intuitive grammars, we need to get away from the whole idea of *syntactic* substitution of one chord for another that underlies Steedman 1984 and Johnson-Laird 1991, and to seek something founded more straightforwardly in a musical semantics or "model theory" for harmony.

- The key to this lies in work by Longuet-Higgins 1962a, 1962b, who showed that the harmonic relation between a pair of notes can be expressed as a vector in a three-dimensional discrete space whose generators are respectively related to integer frequency ratios of two, three, and five, and no others.

- It will be convenient to project this three dimensional space onto two dimensions along the "times two" axis, since this corresponds to the octave.

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Figure 3: (Part of) The Space of Note-names (Longuet-Higgins 1962a,b)

III^{-}	VII^{-}	#IV^{-}	#I	#V	#II	#VI	#III^{+}	#VII^{+}
I^{-}	V^{-}	II^{-}	VI	III	VII	#IV	#I^{+}	#V^{+}
bVI^{-}	bIII^{-}	bVII^{-}	IV	I	V	II	VI^{+}	III^{+}
bIV^{-}	bI^{-}	bV^{-}	bII	bVI	bIII	bVII	IV^{+}	I^{+}
bbII^{-}	bbVI^{-}	bbIII^{-}	bbVII	bIV	bI	bV	bII^{+}	bVI^{+}

Figure 4: (Part of) The Space of Disambiguated Harmonic Intervals

Harmony Theory (Notes)

- In this figure the intervals are disambiguated. The prefix $\sharp$ and $\flat$ roughly correspond respectively to the traditional notions of “augmented” intervals, and to “minor” and/or “diminished” intervals, while the superscripts plus and minus roughly correspond to the “imperfect” intervals.

- This is a musical true fact, which is obscured by modern equally tempered tuning, which equates all positions separated by the Comma of Didymus (yielding the spiral space of standard notation explored by Chew 2001), as well as those related by an augmented seventh (yielding the toroidal space of equal temperament).

Chord Progression

- Musically coherent chord sequences such as the twelve-bar blues have something to with orderly progression to a destination by small steps in this space.
- For example, the basic sequence in Figure 1a, repeated as Figure 2a, is a closed journey around a central I visiting the immediately neighbouring IV and V. Figure $2a'$ makes a jump to the right to II, then returns via V.
- The work of moving the harmonic reference point around in the space is mainly done by dominant seventh chords.

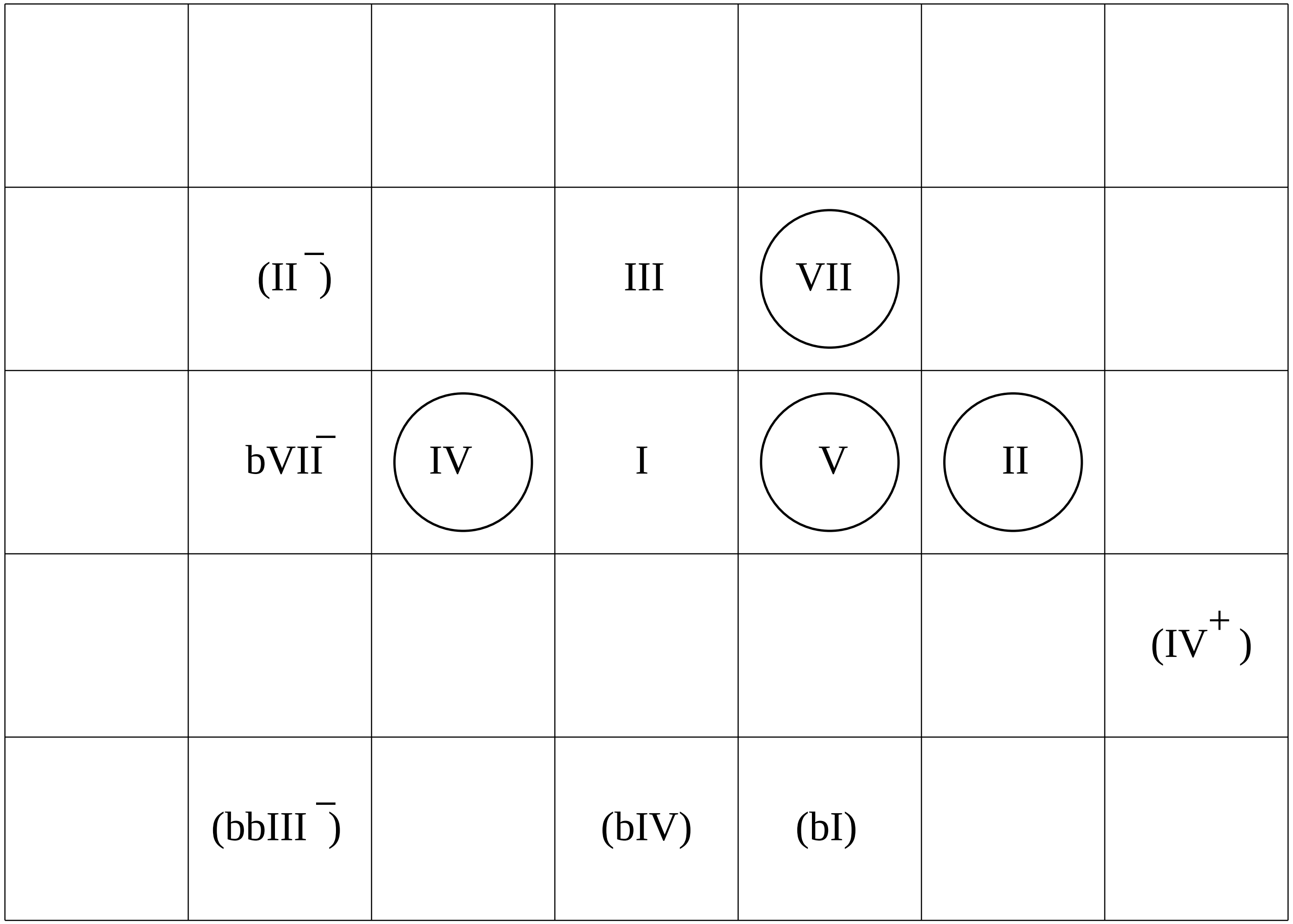


Figure 5: The Dominant Seventh Chord (circles) and its resolution (squares)

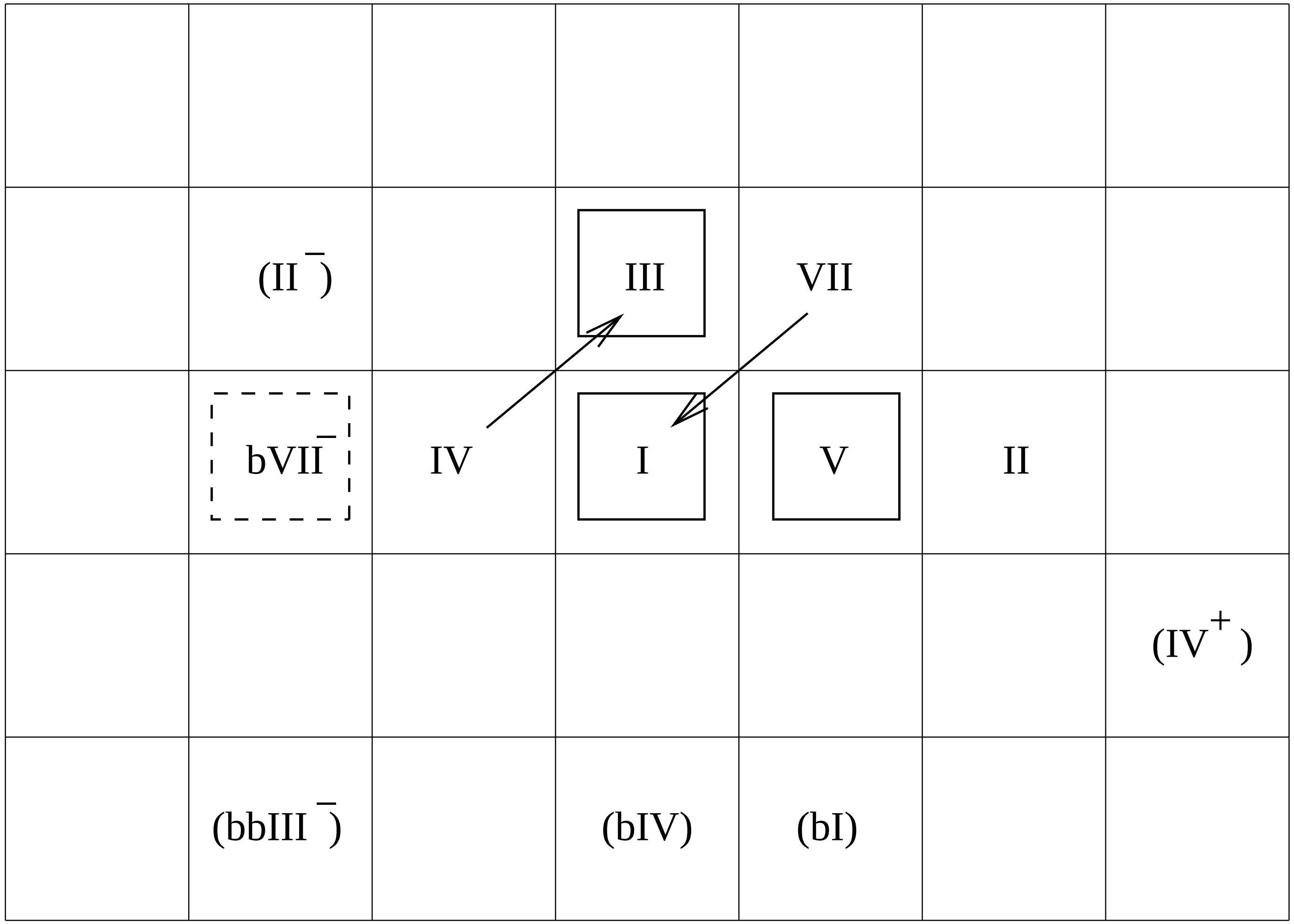


Figure 6: The Dominant Seventh Chord (circles) and its resolution (squares)

The Dominant Seventh

- The representation makes it obvious why the harmonically closest interpretations of the IV and the II are not any of the imperfect or diminished alternatives shown in brackets. It is the addition of the dominant seventh of V, the circled IV, that makes the V chord have a hole in its middle, into which a triad on II (squared II, III, and IV) fits neatly, sharing one note with the first chord, and with the two remaining notes standing in semitone “leading note” relations. (There are voice-leading inplications here.)

- This is a different kind of disambiguation, of individuals in the model, comparable to resolution of pronoun reference in natural language.

- Now we need a syntax that is capable of supporting this semantics for use in a parser.

Combinatory Categorial Grammar for English

- (2) $S \rightarrow NP\ VP$

  $VP \rightarrow TV\ NP$

  $TV \rightarrow \{\text{eats}, \text{drinks}, \ldots\}$

  (3) eats := $(S\backslash NP)/NP$

  (4) *Functional Application:*

  $$a.\quad X/Y \quad Y \quad \Rightarrow \quad X$$

  $$b.\quad Y \quad X\backslash Y \quad \Rightarrow \quad X$$

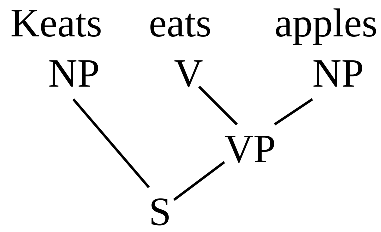
CCG for English

(5) a.

$$\dfrac{\text{Keats}}{NP : keats} \quad \dfrac{\text{eats}}{(S\backslash NP)/NP : \lambda x, y.eats(y, x)} \quad \dfrac{\text{apples}}{NP : apples}$$

$$\dfrac{(S\backslash NP)/NP : \lambda x, y.eats(y, x) \quad NP : apples}{S\backslash NP : \lambda y.eats(y, apples)} >$$

$$\dfrac{NP : keats \quad S\backslash NP : \lambda y.eats(y, apples)}{S : eats(keats, apples)} <$$

b. 

- (The annotations $>$ and $<$ on combinations in a, above, are mnemonic for the rightward and leftward function application rules 4a,b).
- In order to capture linguistics phenomena such as coordination, relativization, intonation structure, and word order in languages other than English, CCG adds syntactic operators related to the Combinators of Schönfinkel and Curry

CCG for English

- The interesting thing about such grammars for present purposes is that the inclusion of these rules allows left-branching analysis of structures like the English clause, which we usually think of as predominantly right-branching.

(6) a.

$$\cfrac{\cfrac{\cfrac{\text{Keats}}{NP : keats}}{S/(S\backslash NP) : \lambda p.p(keats)}{>}\mathbf{T} \quad \cfrac{\text{eats}}{(S\backslash NP)/NP : \lambda x,y.eats(y,x)}}{S/NP : \lambda x.eats(keats,x)}{>}\mathbf{B} \quad \cfrac{\text{apples}}{NP : apples}$$

$$\overline{\quad S : eats(keats, apples) \quad}{>}$$

- As a result they make incremental interpretation easy in language and music.

A Categorial Chord Grammar

1*a*.	X	:=	I_X
1*b*.	Xm	:=	$I_X m$
2*a*.	X	:=	$V_X \backslash V_X$
2*b*.	Xm	:=	$V_X m \backslash V_X m$
3*a*.	Xm^7	:=	$I_X m^7 / IV_X^7$
3*b*.	X^7	:=	$I_X^7 / IV_X(m)^7$
4.	Xm^7	:=	$VII_X m^7 / VII_X m^7$
5.	Xm	:=	$\flat VII_X m \backslash \flat VII_X m$
6.	X∘7	:=	$\flat V_X / \flat V_X$
			$\flat II_X / \flat II_X$
			$\flat VII_X m^7 / \flat VII_X m^7$

Figure 7: A Categorial Chord Grammar

- I_X, V_X, etc. are I, V etc. relative to the root X on the left of the :=.
- The option brackets round the minor annotation (m) mean that if the basic chord category is minor then so is the categorial type.
- Most categories are simply the identity function, but 3a and 3b do the real work of elaborating the perfect cadence. The reason they are I^7/IV^7 rather than I^7/IV is to do with recognizing the end of the cadence, and is dealt with below.
- We add a pair of trivial syncategorematic rules resembling coordination that make sequences of Xs into a single X. These have the property of passing the ^{7} marker to the rightmost daughter (brackets here mean the $(^7)$ is optional.

(7) $X(m) \quad X(m) \quad \Rightarrow \quad X(m)$ $\qquad (< \& >)$

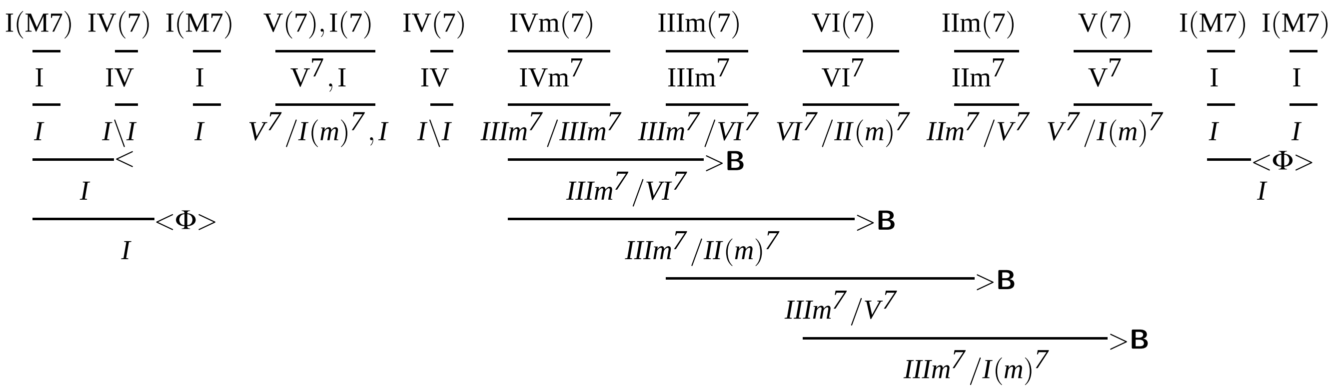
$X(m)(^7) \quad X(m)^7 \quad \Rightarrow \quad X(m)^7$ $\qquad (< \& >)$

- This notation can easily be augmented to enforce the condition that the combination of an X/Y occupying a bars with a Y occupying b bars yields an X occupying $a+b$ bars. This detail is omitted.

A Derivation in the Categorial Grammar

- Together with the same “phonological spelling” rules as before (1), and function composition and type-raising as well as function application, this grammar gives rise to (incomplete) derivations like the following for the chord sequence *c* in Figure 1:

(8)



The Categorial Grammar of Authentic Cadences (Contd.)

- Unlike Steedman 1994, this fragment does not work by substitution on a previously prepared skeleton.

- It is still incomplete, in that it does not yet specify the higher levels of analysis that stitch the sequences of cadences together into canonical forms like twelve-bars, and variations on *I Got Rhythm*.

- Stochastic POS tagging techniques are likely to do very well at disambiguating homophones like $\mathrm{X}(7)$ chords. ($\mathrm{X}(7)$ is likely to be X^7 if followed by IV_{X}, X if followed by V_{X}, etc.)

- Just as in the linguistic grammar, we can associate a semantic interpretation with categories, which the combinatory rules will "project" onto derivational structure, as follows:

The Categorial Grammar with Semantics

1a.	X	:=	$I_X : X$
1b.	Xm	:=	$I_X m : X$
2a.	X	:=	$V_X \backslash V_X : \lambda x.x$
2b.	Xm	:=	$V_X m \backslash V_X m : \lambda x.x$
3a.	Xm^7	:=	$I_X m^7 / IV_X^7 : \lambda x.leftonto(x)$
3b.	X^7	:=	$I_X^7 / IV_X(m)^7 : \lambda x.leftonto(x)$
4.	Xm^7	:=	$VII_X m^7 / VII_X m^7 : \lambda x.x$
5.	Xm	:=	$\flat VII_X m \backslash \flat VII_X m : \lambda x.x$
6.	X∘7	:=	$\flat V_X / \flat V_X : \lambda x.x$
			$\flat II_X / \flat II_X : \lambda x.x$
			$\flat VII_X m^7 / \flat VII_X m^7 : \lambda x.x$

Figure 8: The Categorial Grammar with Semantics

Semantic Derivation of the Extended Cadence

(9) …

IVm(7)	IIIm(7)	VI(7)	IIm(7)	V(7)	I(M7)	I(M7)
IVm^7	IIIm^7	VI^7	IIm^7	V^7	I	I
$IIIm^7/IIIm^7 : \lambda x.x$	$IIIm^7/VI^7 : \lambda x.leftonto(x)$	$VI^7/II(m)^7 : \lambda x.leftonto(x)$	$IIm^7/V^7 : \lambda x.leftonto(x)$	$V^7/I(m)^7 : \lambda x.leftonto(x)$	$I : I$	$I : \lambda x.x$

$$\frac{IIIm^7/IIIm^7 : \lambda x.x \quad IIIm^7/VI^7 : \lambda x.leftonto(x)}{IIIm^7/VI^7 : \lambda x.leftonto(x)} >\mathbf{B}$$

$$\frac{IIIm^7/VI^7 : \lambda x.leftonto(x) \quad VI^7/II(m)^7 : \lambda x.leftonto(x)}{IIIm^7/II(m)^7 : \lambda x.leftonto(leftonto(x))} >\mathbf{B}$$

$$\frac{IIIm^7/II(m)^7 \quad IIm^7/V^7 : \lambda x.leftonto(x)}{IIIm^7/V^7 : \lambda x.leftonto(leftonto(leftonto(x)))} >\mathbf{B}$$

$$\frac{IIIm^7/V^7 \quad V^7/I(m)^7 : \lambda x.leftonto(x)}{IIIm^7/I(m)^7 : \lambda x.leftonto(leftonto(leftonto(leftonto(x))))L} >\mathbf{B}$$

$$\frac{I : I \quad I : \lambda x.x}{I : I} <\Phi>$$

- The cadential category $IIIm^7/I(m)^7 : \lambda x.leftonto(leftonto(leftonto(leftonto(x))))$ cannot yet combine with the following $I : I$ since it requires I^7. If it did, it would yield exactly the semantics we want:

(10) $IIIm^7 : leftonto(leftonto(leftonto(leftonto(I))))$

- But the syntactic type $IIIm^7$ isn't the right name for that.

A Derivation (Contd.)

- A cadence requires an origin as well as a destination.
- Instead of just applying an extended cadence to its target, we will give the target a higher-order type that labels the result explicitly as $I_X \backslash I_X$, the category of a non-initial cadential modifier of I_X, via the following rule reminiscent of type-raising:

(11) 7. $X \quad \rightarrow \quad (I_X \backslash I_X) \backslash (Y^7 / I_X^7)$

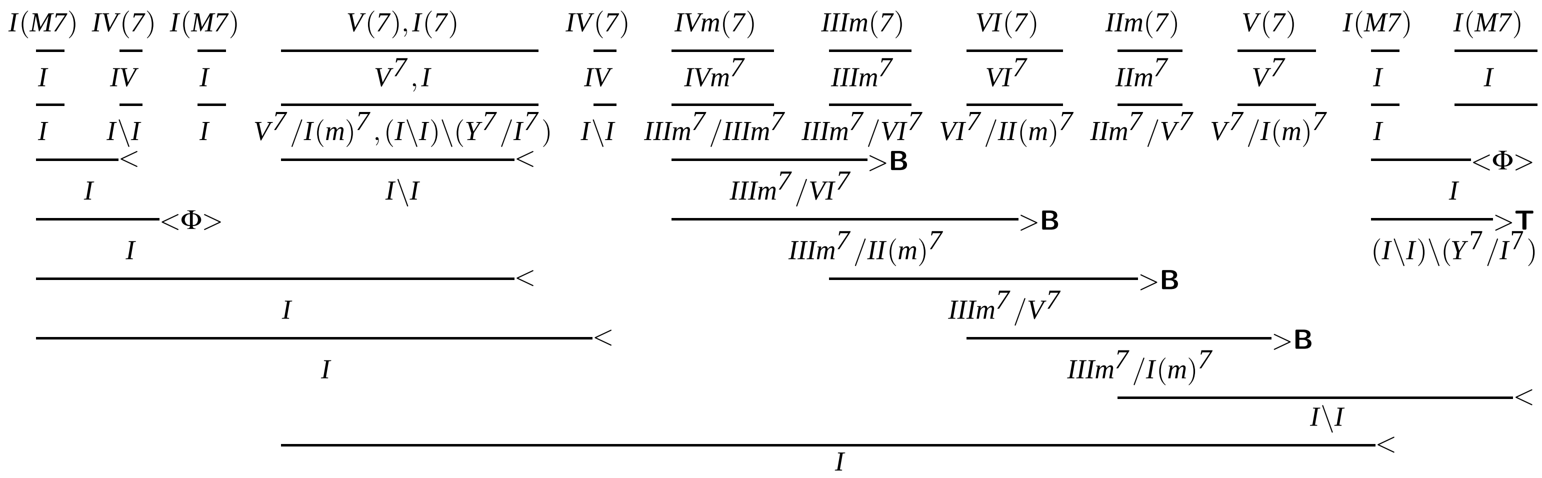
- Semantically we can think of the rule as follows

(12) 7. $X : origin \quad \rightarrow \quad (I_X \backslash I_X) \backslash (Y^7 / X^7) : \lambda cadence.\lambda origin.origin + cadence(origin)$

A Derivation

- With this category we can complete the earlier derivation as follows:

(13)



- The interpretation (whose derivation is suggested as an exercise) is as follows

(14) $(I + (leftonto(I))) + (leftonto(leftonto(leftonto(leftonto(I)))))$

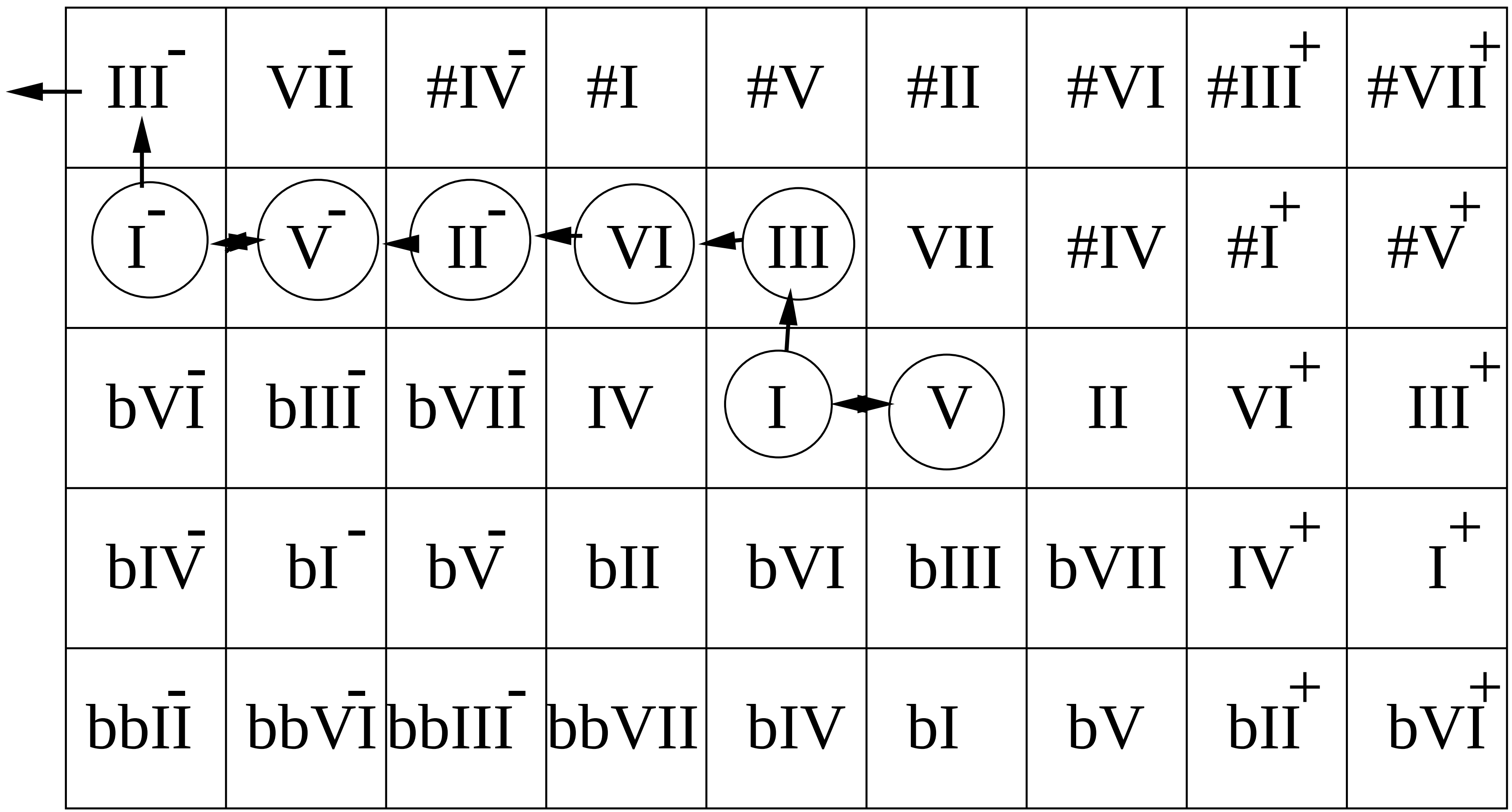


Figure 9: The Denotation of an extended Authentic Cadence (*Basin St. Blues*)

A Model for the Interpretation

- This denotation, which corresponds to Figure $2a'''$, is interesting, because it takes a step up to III, then proceeds via leftward steps to end up on I^-.
- I^- is musically distinct from the original I, and if perfectly intoned (as opposed to being played on an equally tempered keyboard), would differ from the original in a ratio of 80:81.
- So it is only practicable to play this sort of music in Equal Temperament.

Extending the Grammar to the Plagal Cadence

- It is a prediction of the theory that the plagal ($IV - I$) cadence will be elaborated in a similar way, to give sequences which are the mirror image of the authentic cadence, with a "right-onto" semantics.

- We can do this by introducing the following two categories parallel to 3a and 3b to replace 2a and 2b:

$$
\begin{array}{lll}
2'a. & Xm & := \quad I_X m / V_X : \lambda x.rightonto(x) \\
2'b. & X & := \quad I_X / V_X(m) : \lambda x.rightonto(x)
\end{array}
$$

Figure 10: The Plagal Cadence Categories

- While the plagal cadence is less commonly exploited than the authentic, this prediction is correct: "Hey Joe" (Hendrix, 1965) is an exact plagal spatial mirror image of figure 9:

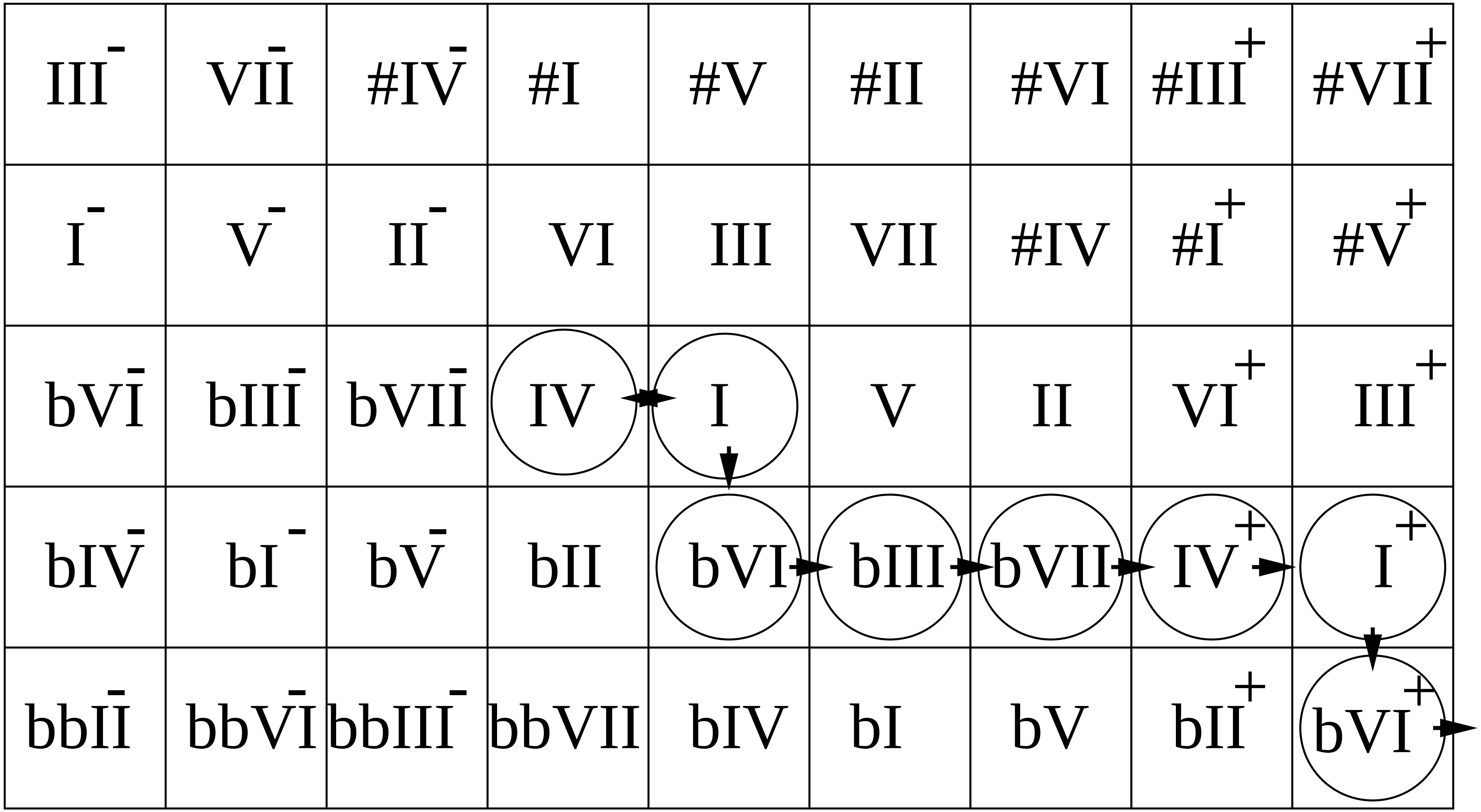


Figure 11: The Denotation of an Extended Plagal Cadence (*Hey Joe*)

Conclusion

- The grammar now interprets twelve-bar and other sequences as being made up of cadences.
- An important result from the point of view of psychological plausibility is that the derivation that produces this very orthodox right branching cadential semantics is predominately *left* branching.
- To that extent, it is also semantically *incremental*, delivering an interpretable result at each reduction, more or less chord by chord.

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