Ants and Agents: Global Complexity from Local Simplicity

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Credit

Based on an original publication:

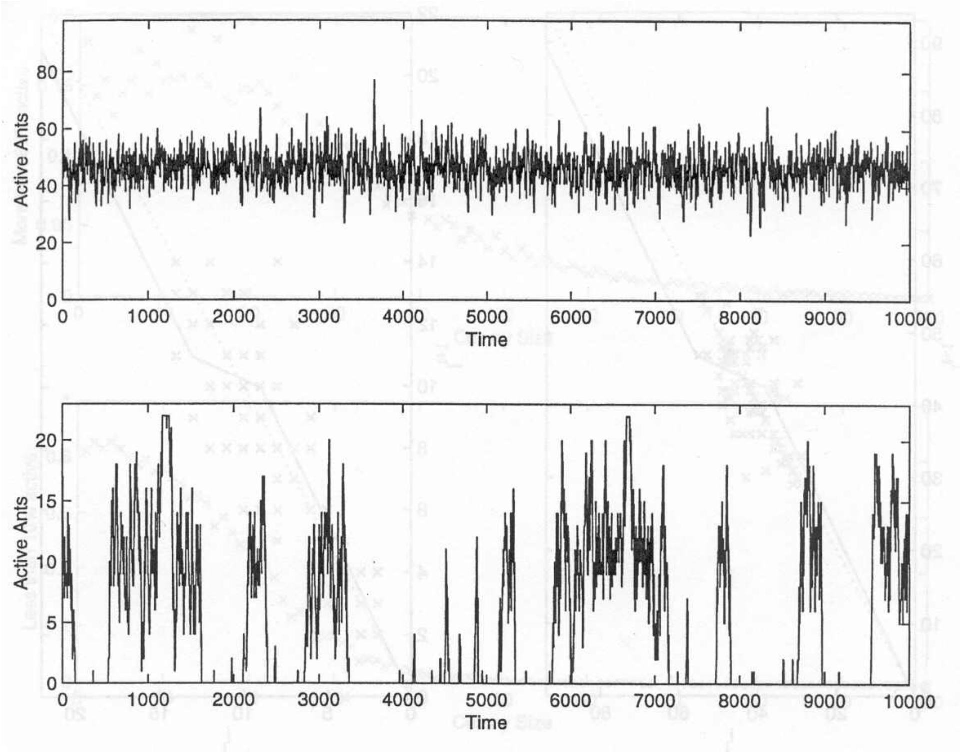
Ants and Agents: a Process Algebra Approach to Modelling Ant Colony Behaviour, D.J.T. Sumpter, G.B. Blanchard and D.S. Broomhead, Bulletin of Mathematical Biology, **63**(2001).

Introduction

- What's the issue?
- Some models of ants
- Mean difference equations as an approximation
- MDE and PEPA?

What's going on?

Plot of activity in: (Top) large colonies and (bottom) small colonies of ants.



The Issues

- Do not doubt the force of the state-space explosion!

  $\Rightarrow$ complex global dynamics can be generated from very simple local dynamics — if there are sufficient interacting agents

- On a technical level — paper uses probabilistic process algebra, WSCCS, for:

  1. specification for simulations
  2. Markov chain analysis
  3. mean difference equation techniques

Active and Passive Ants

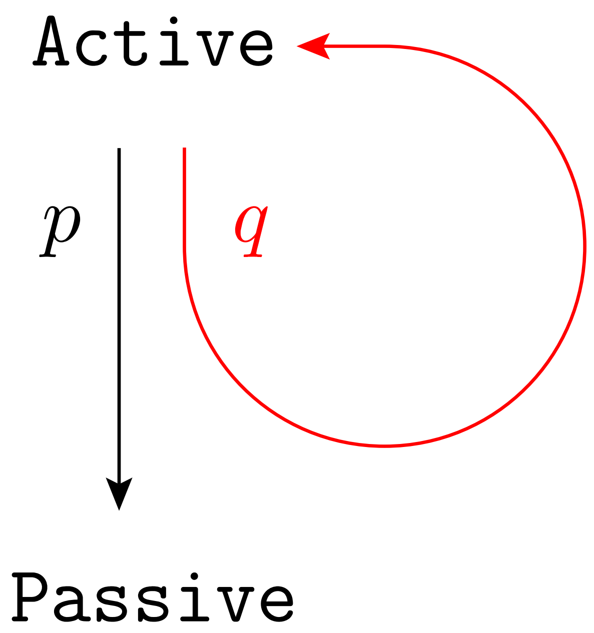
- n agents

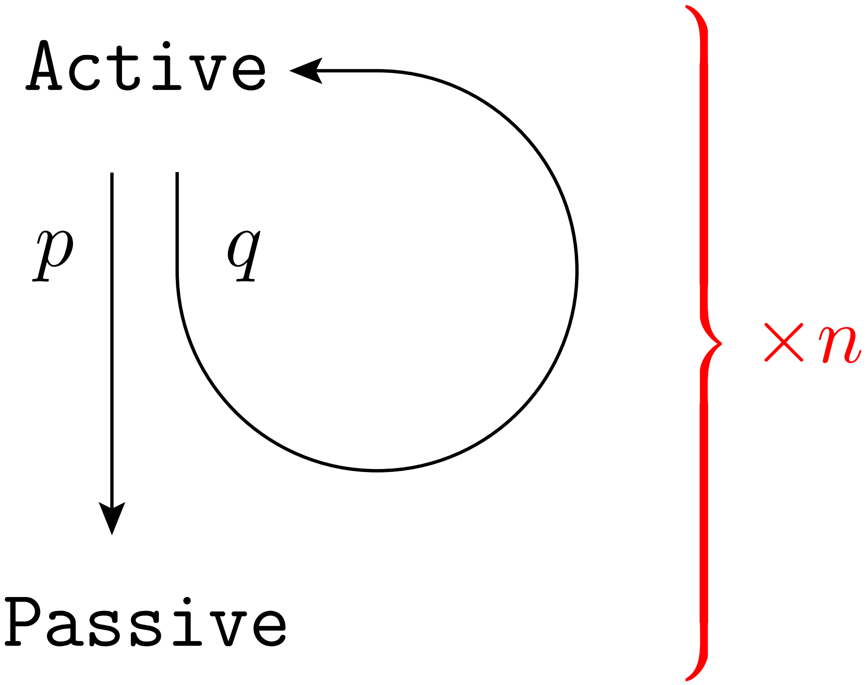
- Active agent can:

  1. go passive with probability p
  2. stay active with probability q

- Passive agents remain passive

$$\begin{tikzcd}
\texttt{Active} \arrow[d, "p"'] \\
\texttt{Passive}
\end{tikzcd}$$


Active
p
q
Passive

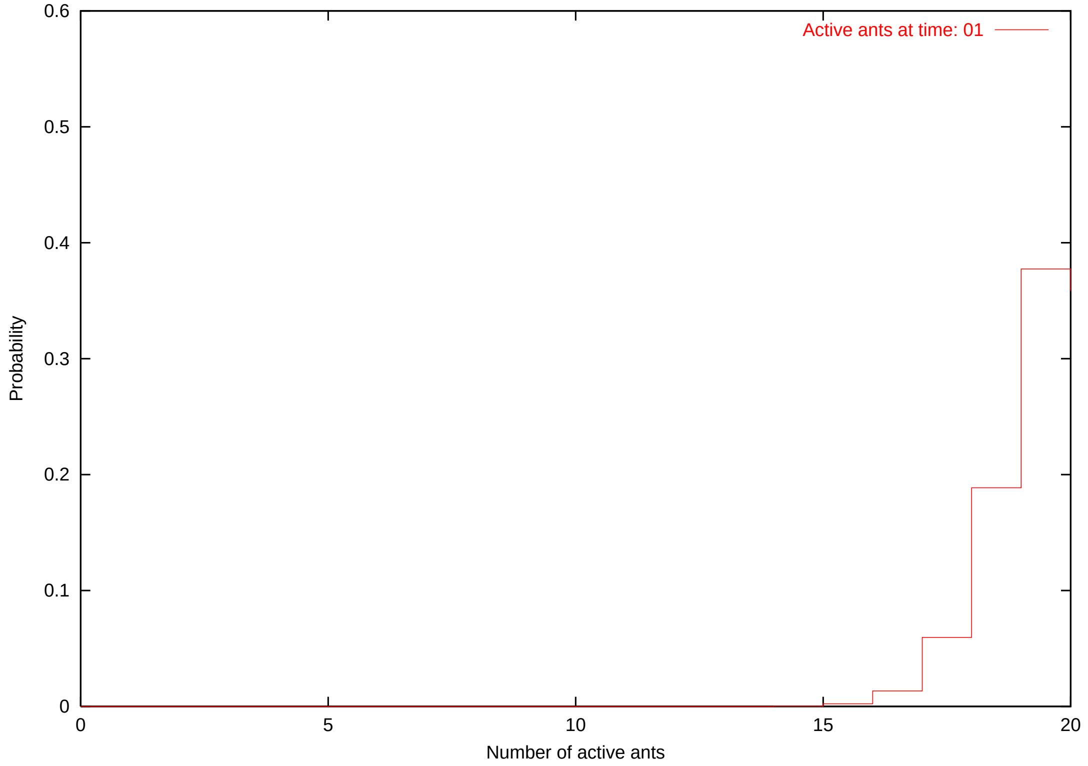

Active
p
q
Passive
$\times n$

WSCCS Description

$$
\begin{aligned}
\texttt{Active} &\equiv p : \surd.\texttt{Passive} + q : \surd.\texttt{Active} \\
\texttt{Passive} &\equiv 1 : \surd.\texttt{Passive} \\
\\
\texttt{Colony}_n(i) &\equiv \prod^{i} \texttt{Active} \times \prod^{n-i} \texttt{Passive}
\end{aligned}
$$

Probability Mass Function of Activity

System 1: $p = 0.05$, $q = 0.95$, $n = 20$, $1 \leq t \leq 71$.



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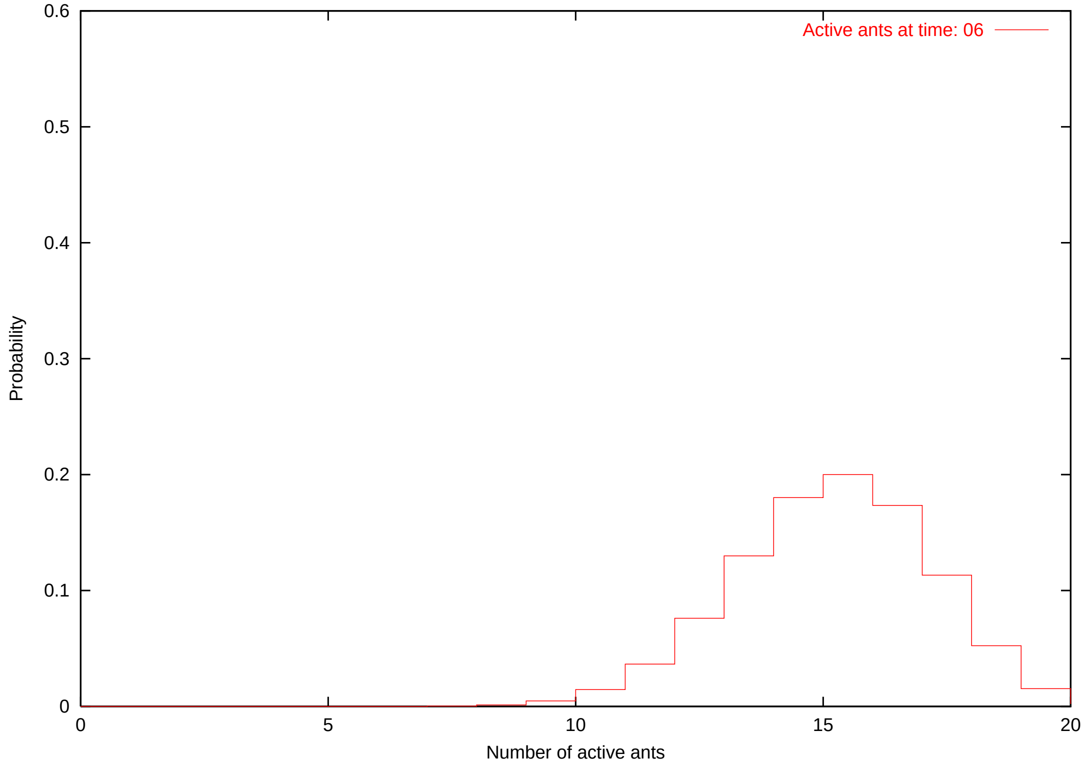

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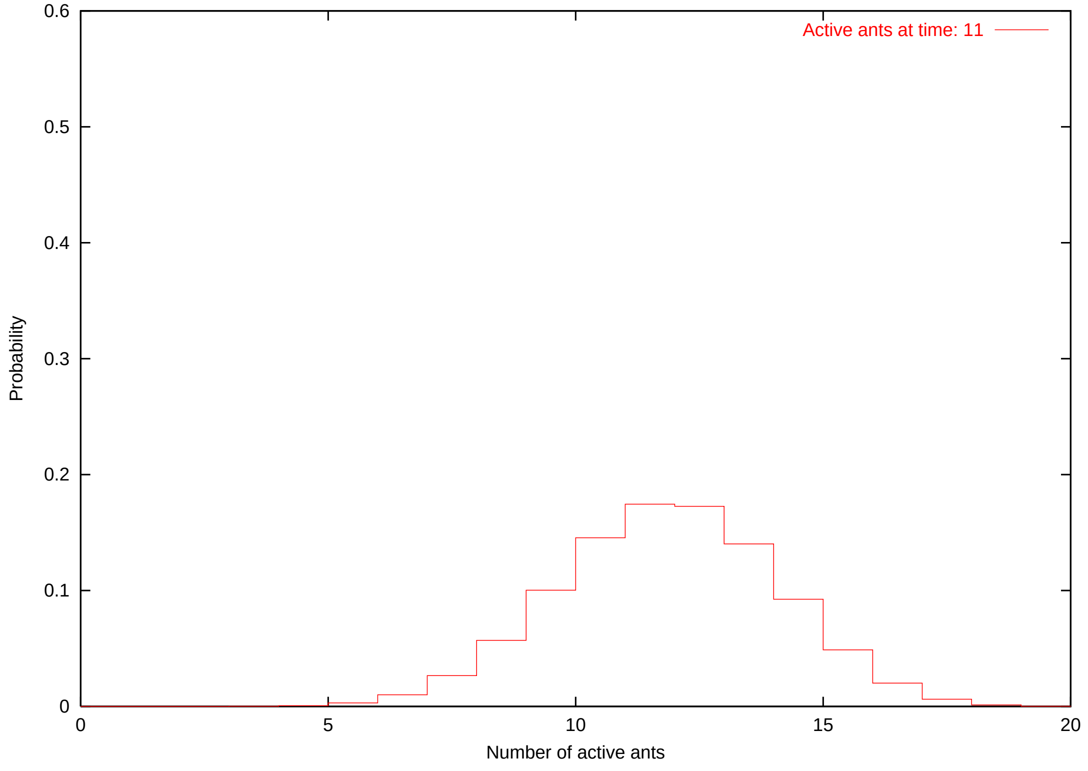

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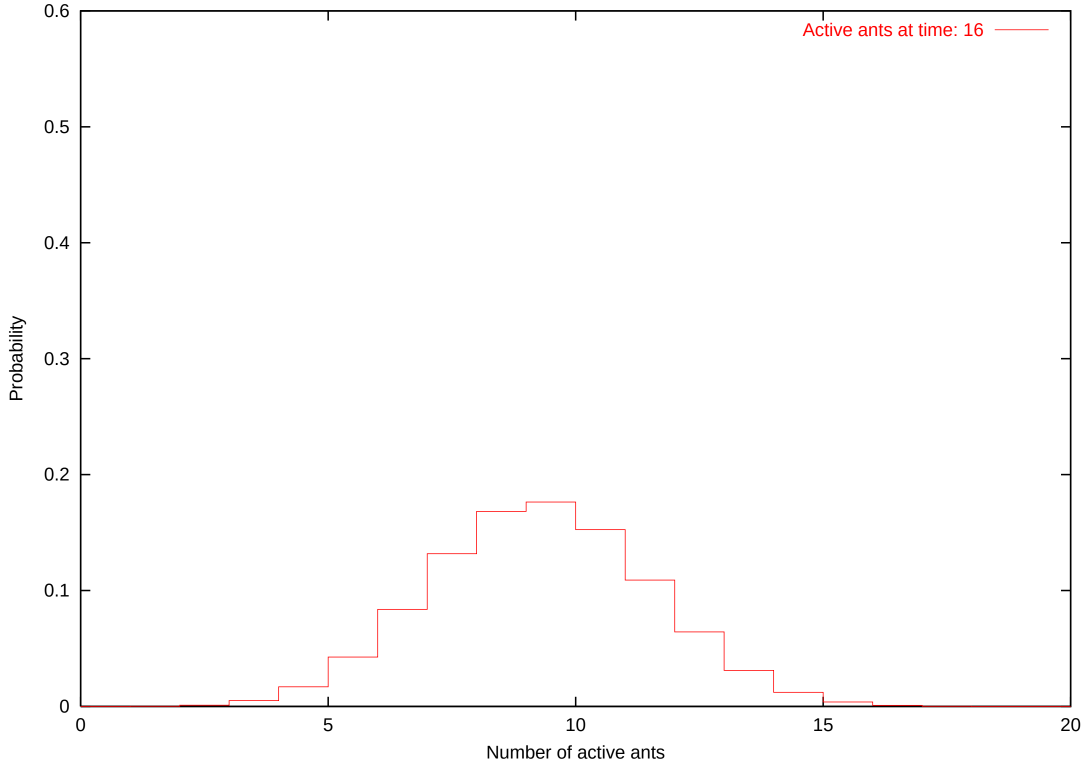

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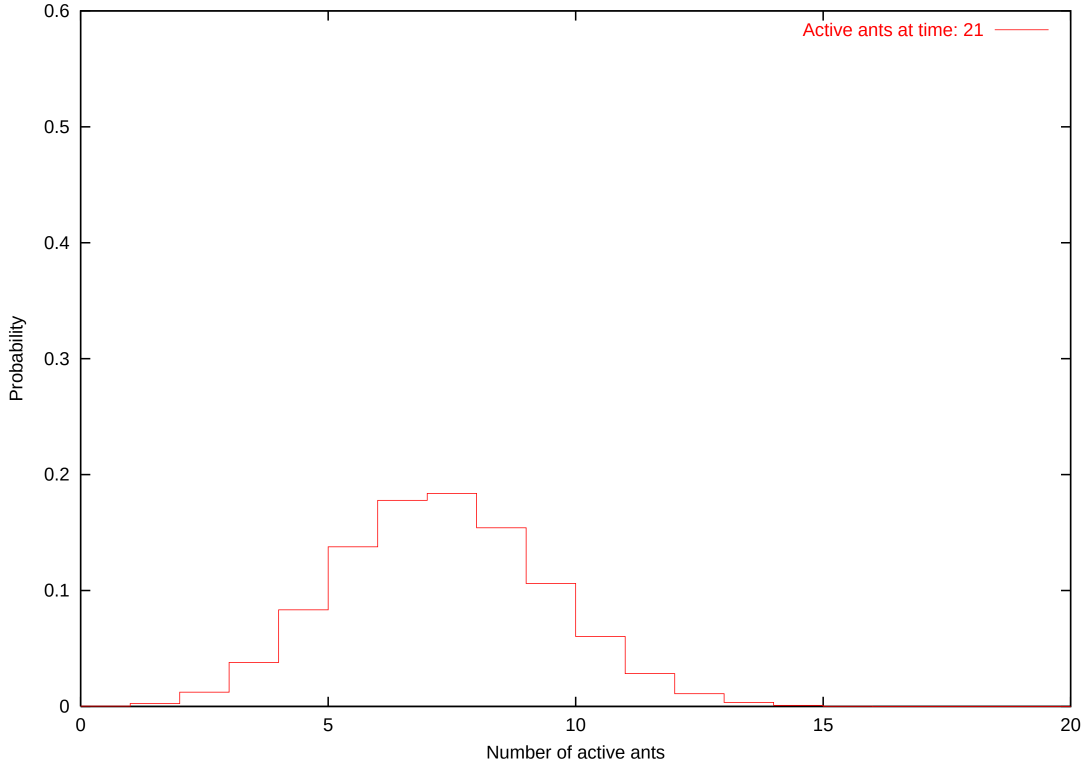

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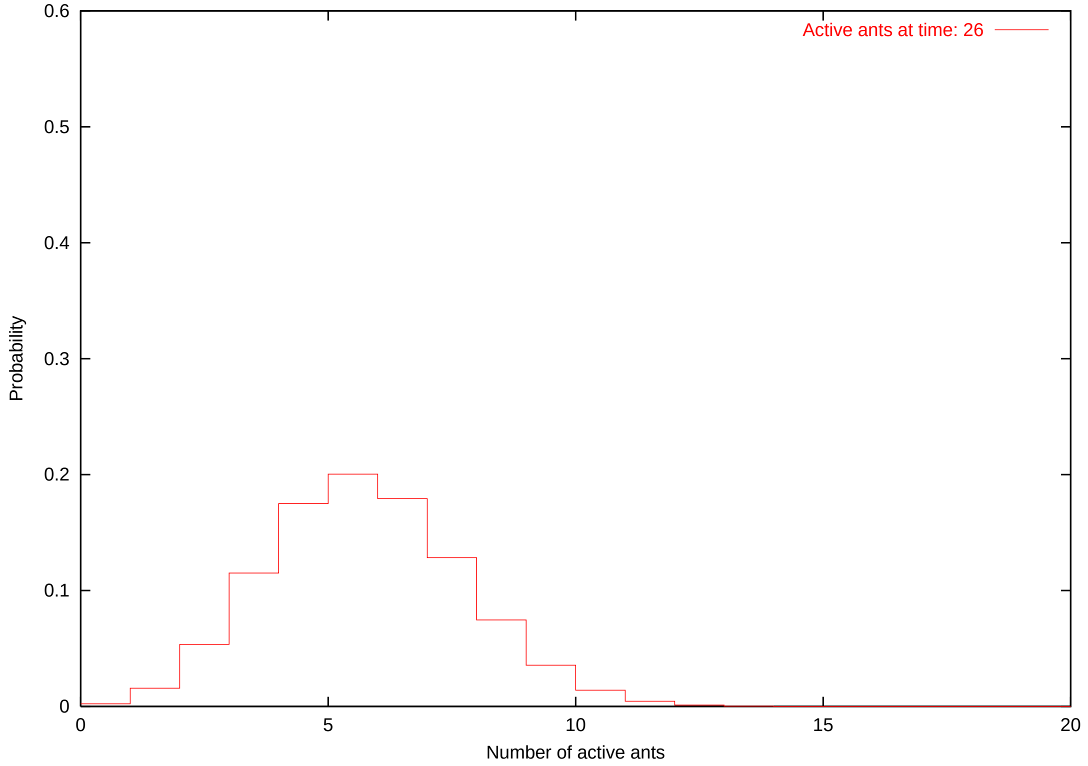

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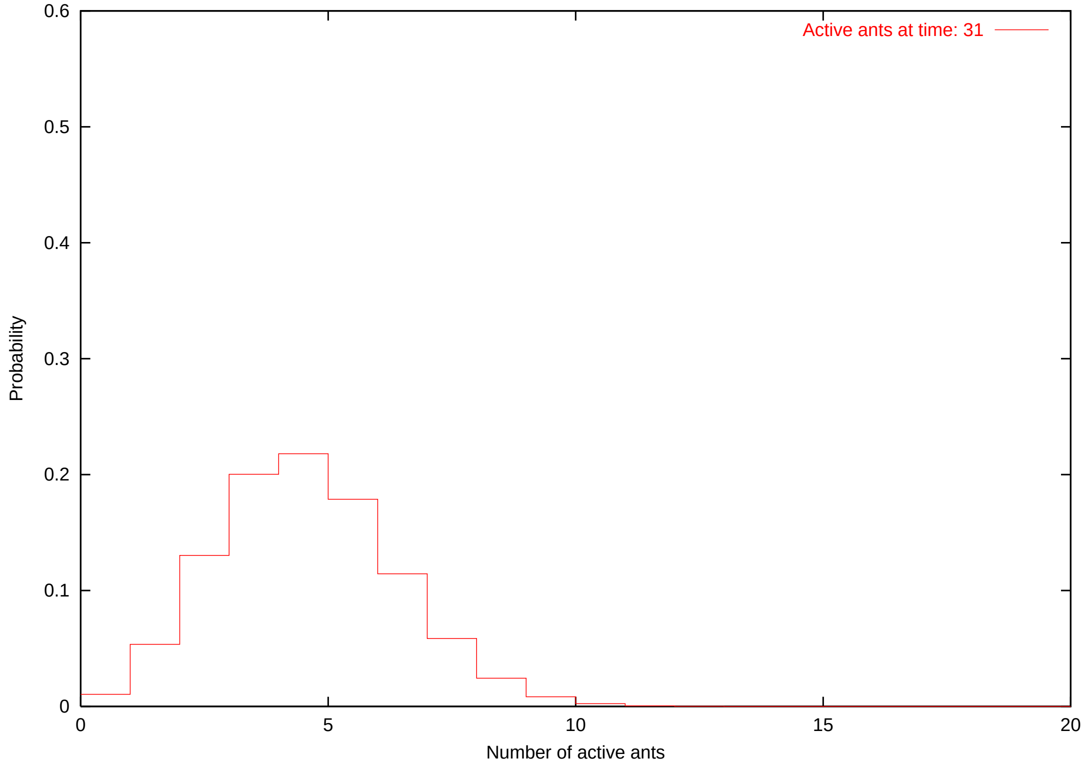

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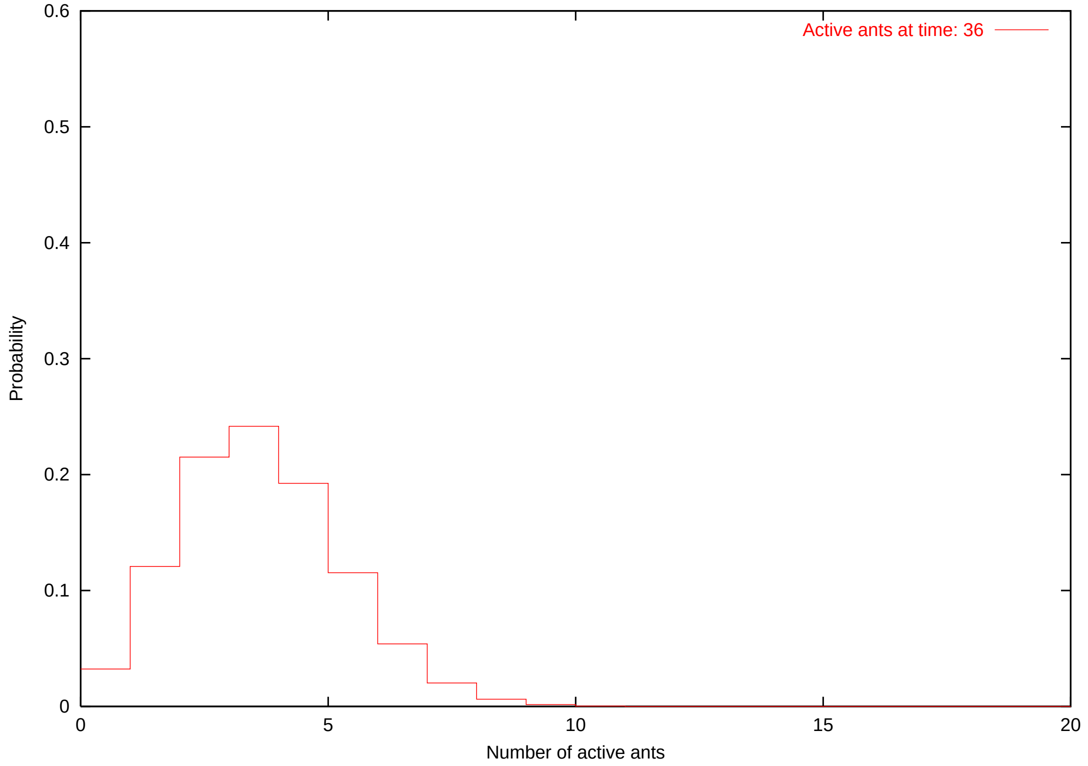

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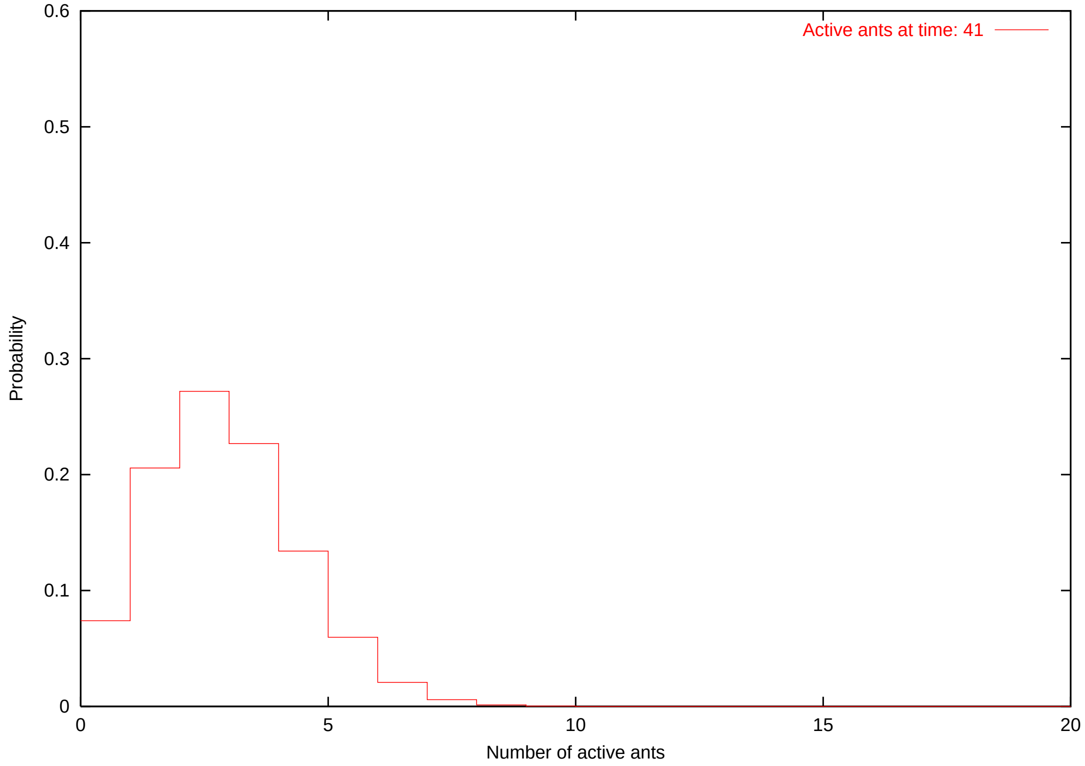

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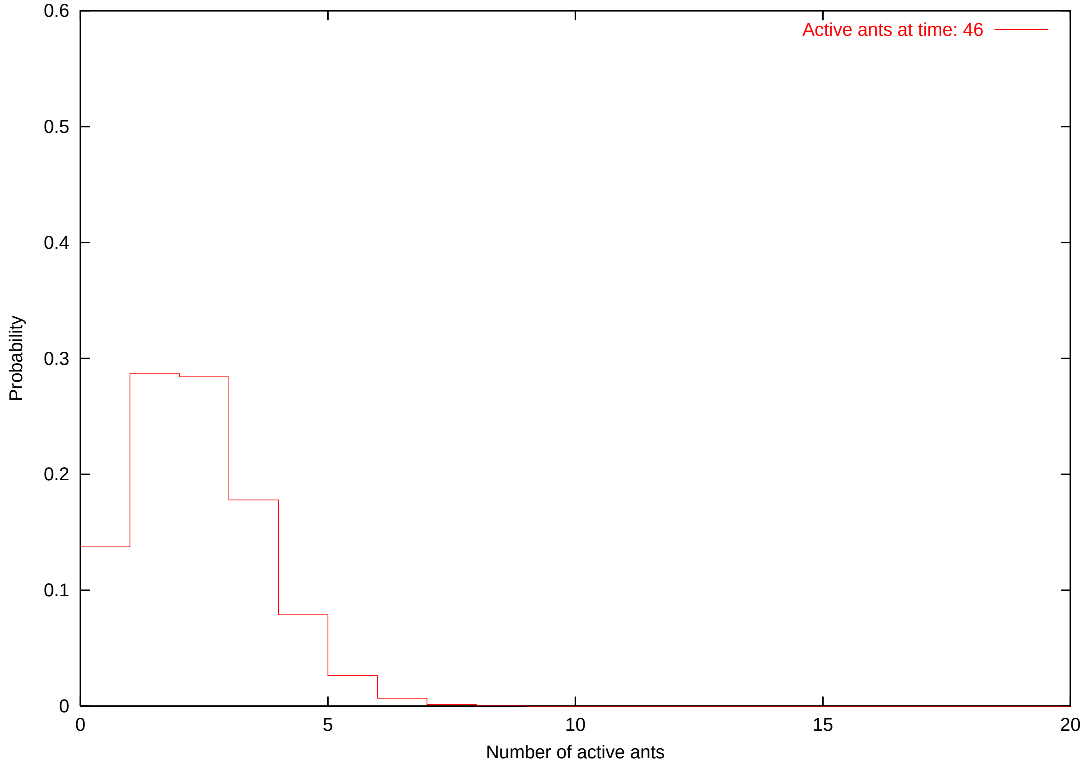

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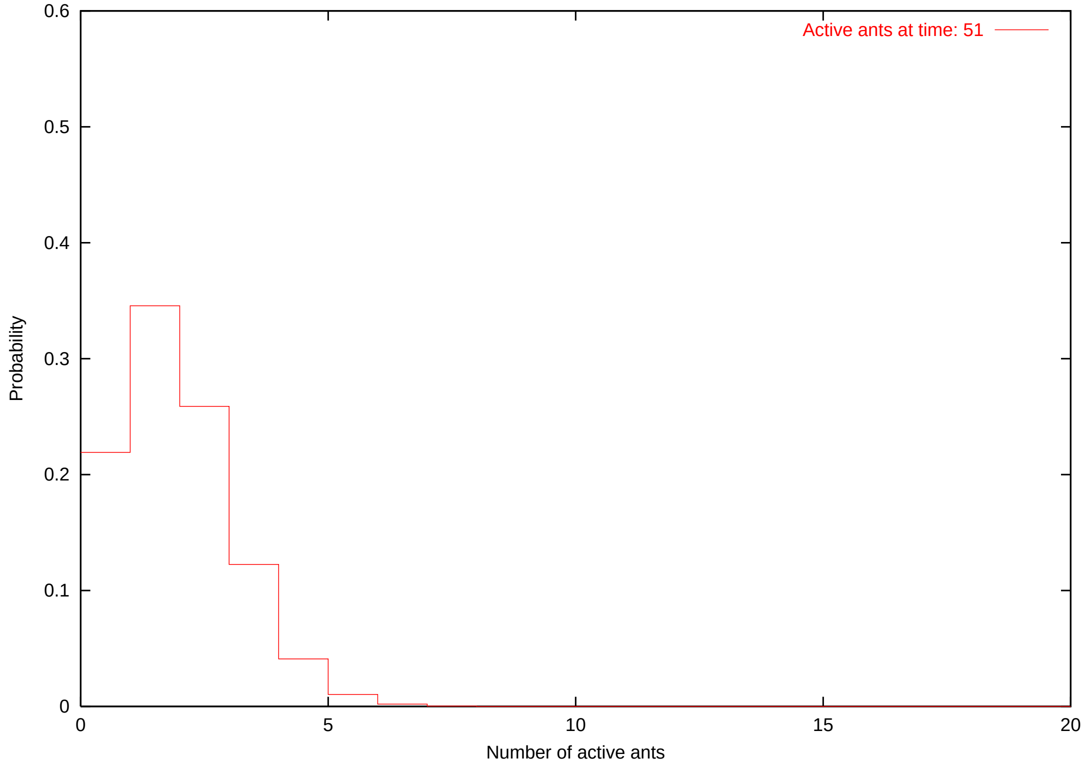

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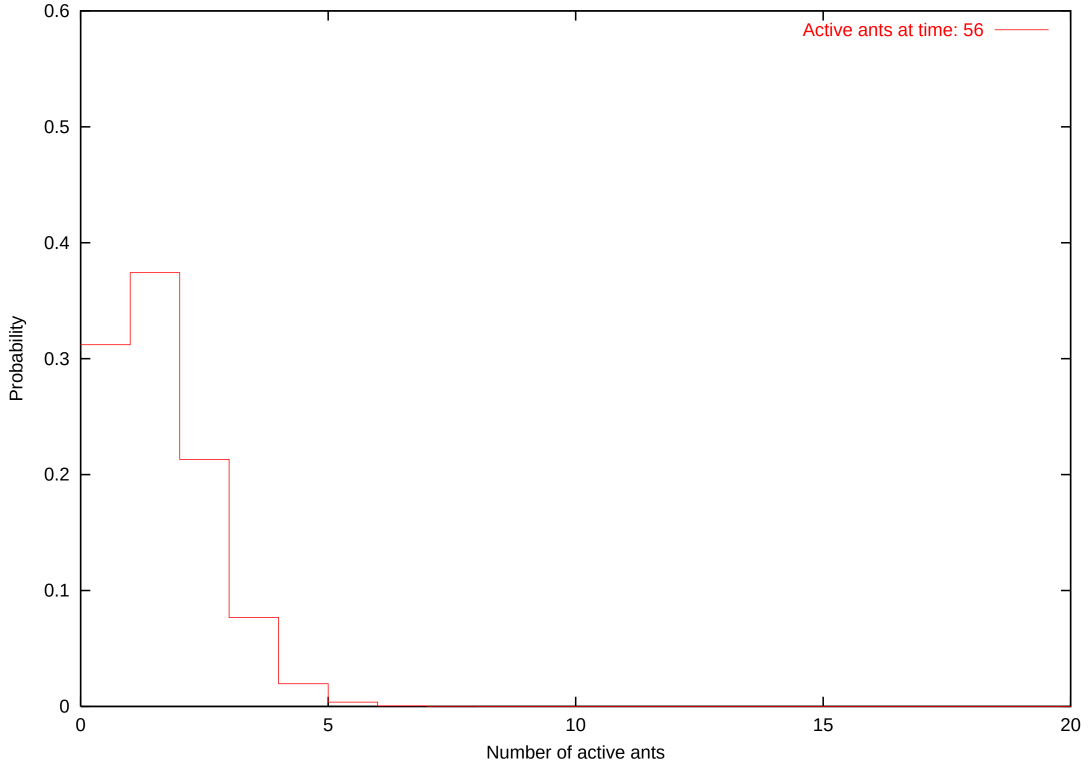

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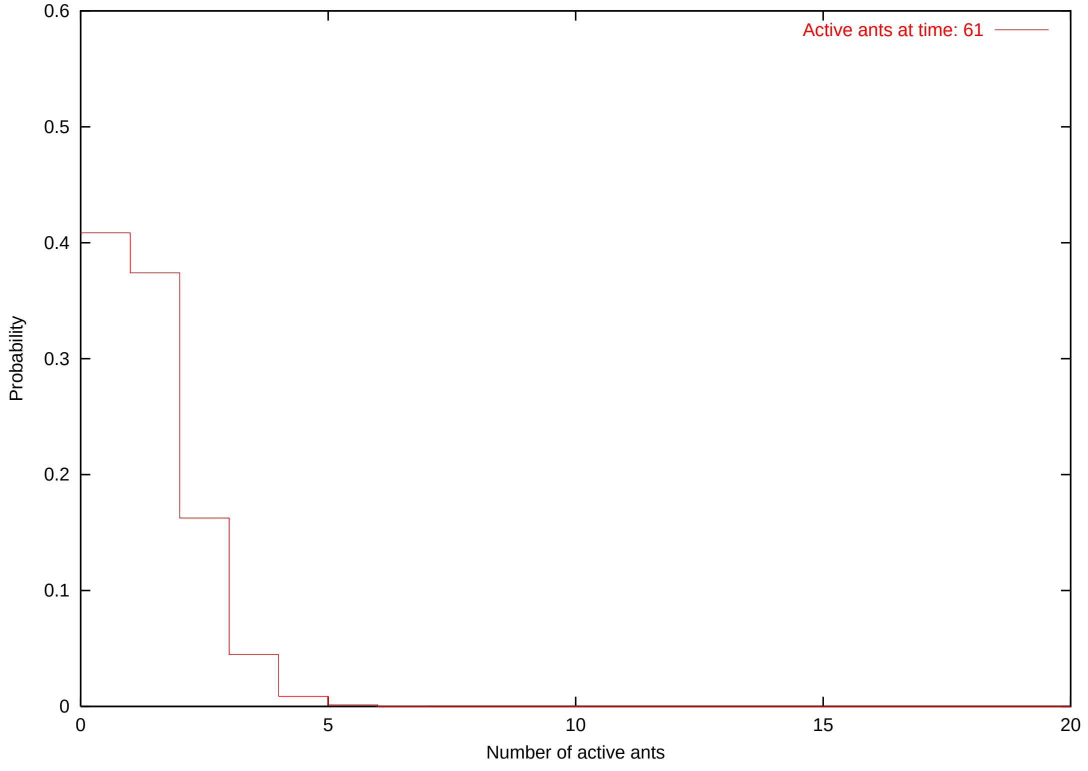

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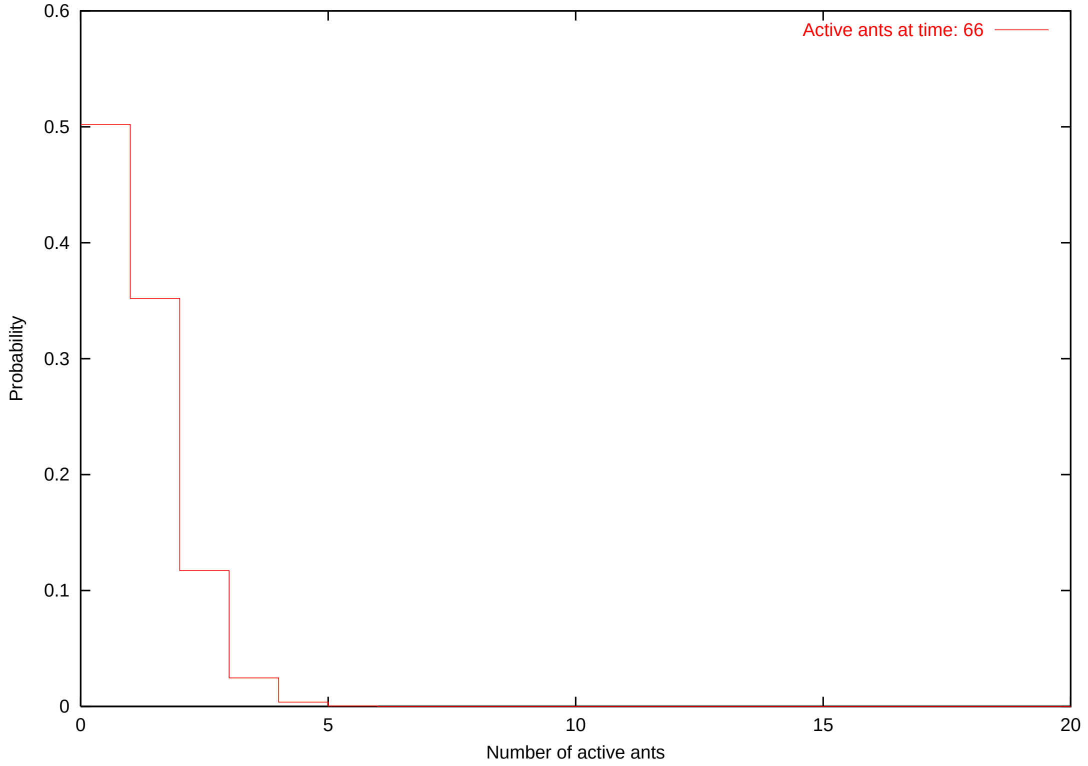

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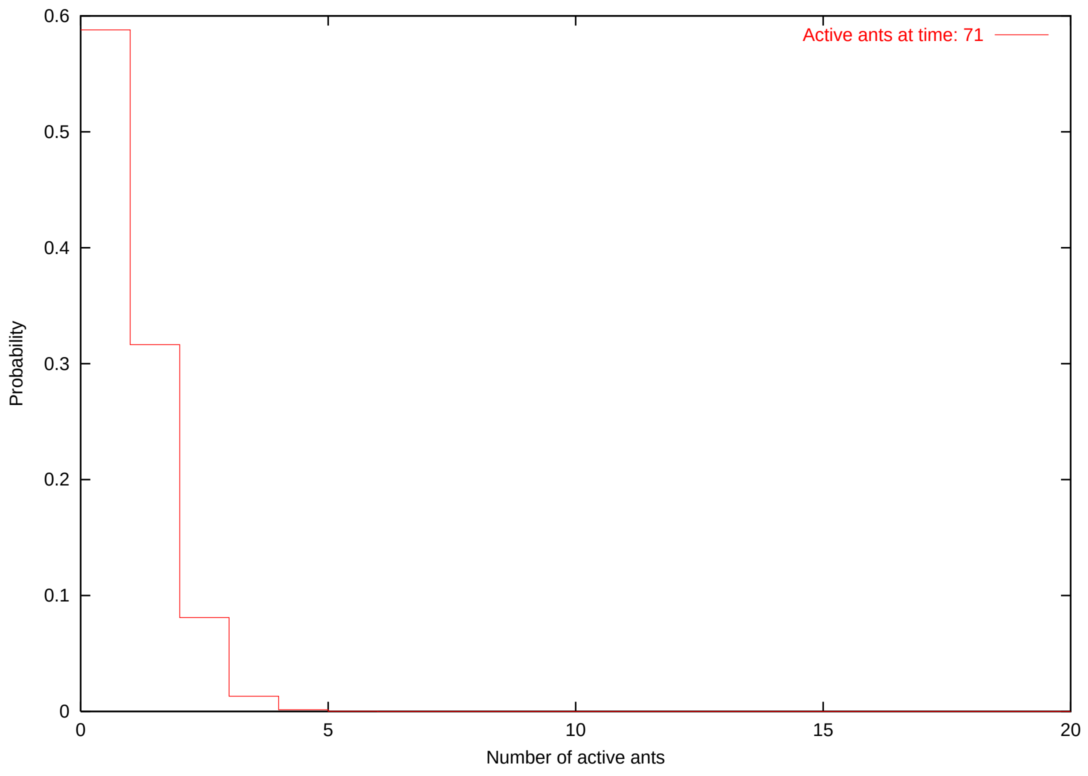


Ant Agent Model (I)

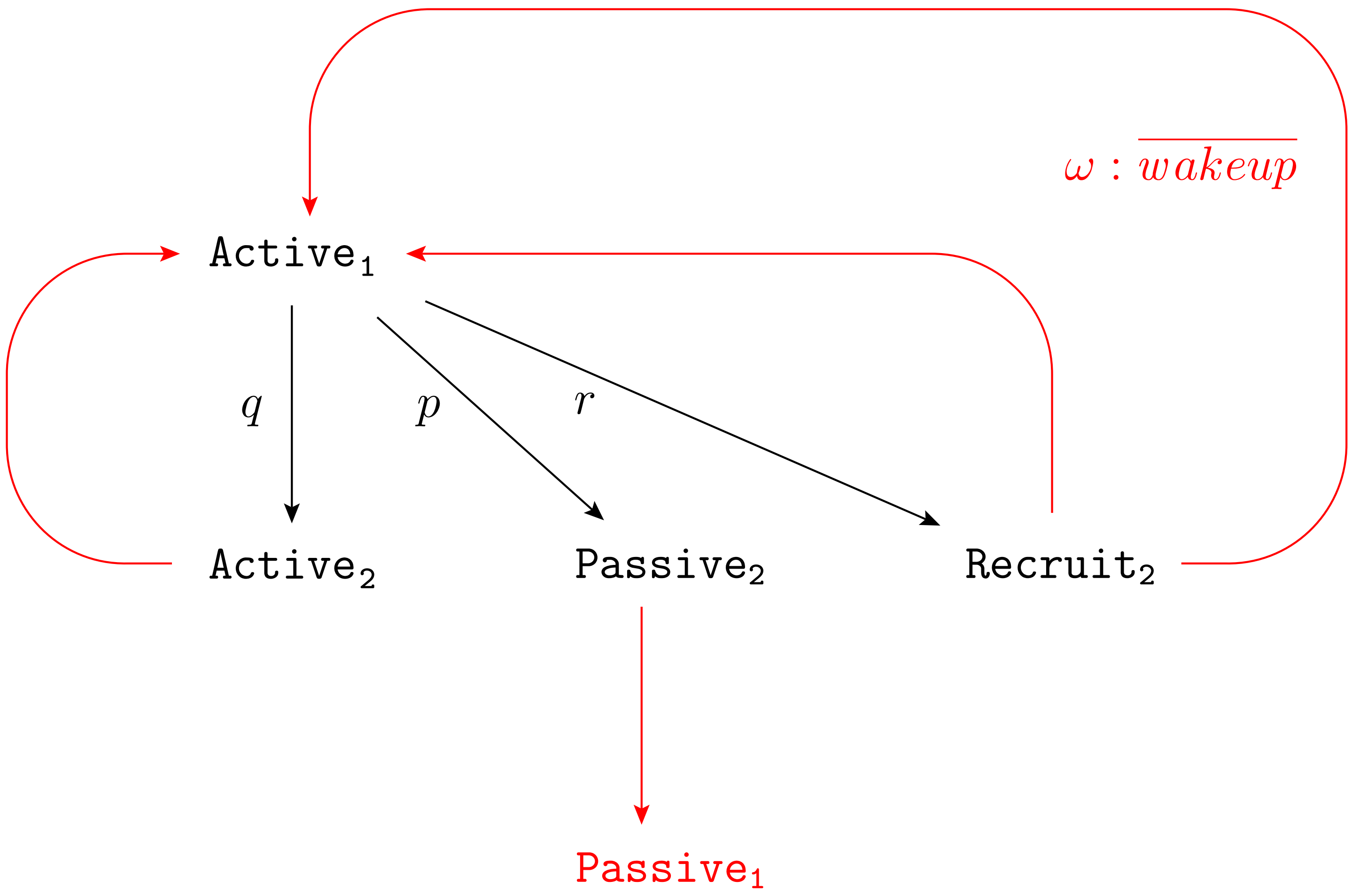
- Communication between ants introduced
- Active ants can now:
  1. remain active
  2. go passive
  3. become a recruiting ant
- Now passive ants can be woken up by recruiting ants

$\Rightarrow$ population does not just decline — it can fluctuate

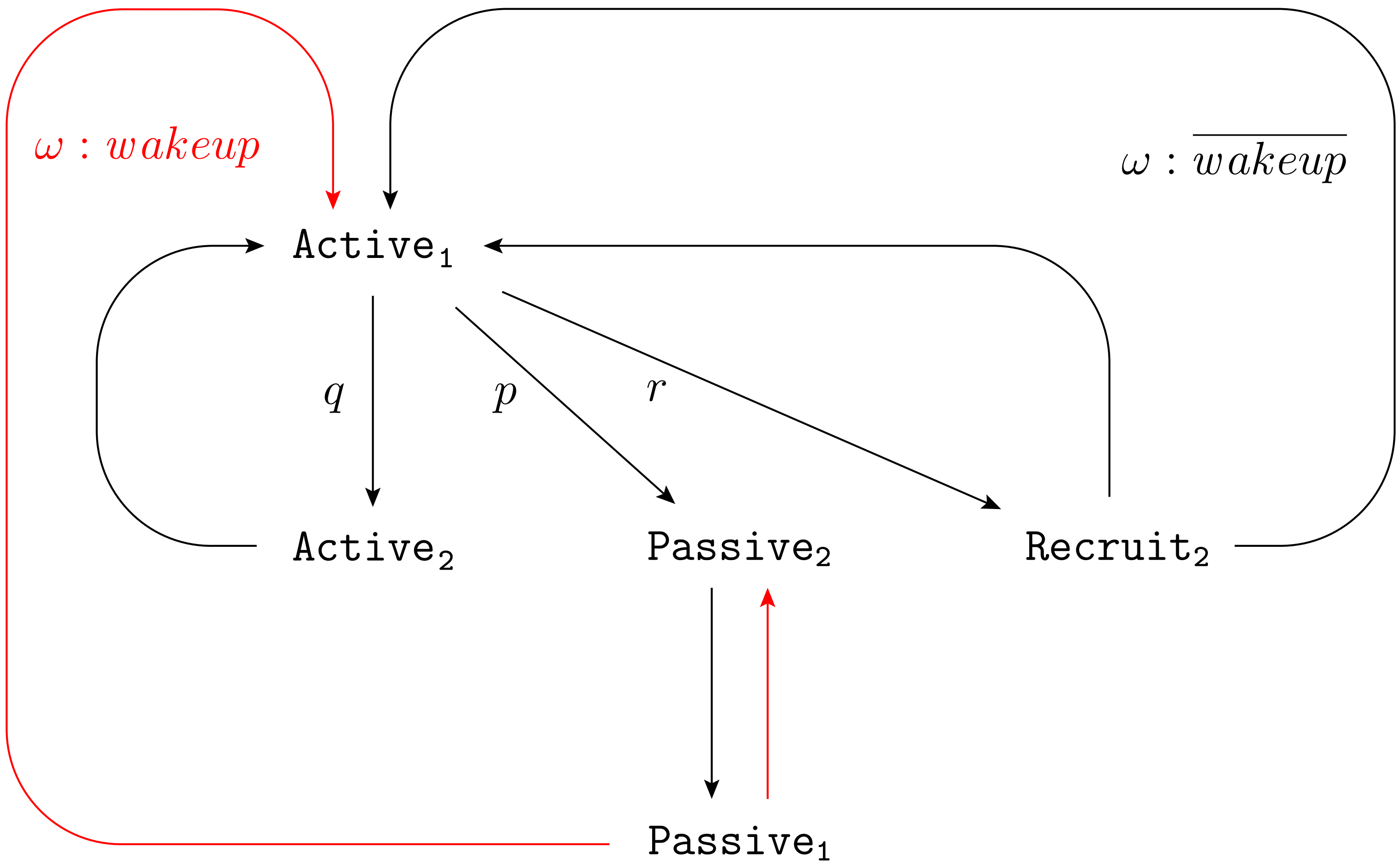
WSCCS Description

$$
\begin{aligned}
\mathtt{Active}_1 &\equiv p : \surd.\mathtt{Passive}_2 + q : \surd.\mathtt{Active}_2 + r : \surd.\mathtt{Recruit}_2 \\
\mathtt{Active}_2 &\equiv 1 : \surd.\mathtt{Active}_1 \\
\mathtt{Recruit}_2 &\equiv \omega : \overline{\mathtt{wakeup}}.\mathtt{Active}_1 + 1 : \mathtt{Active}_1 \\
\\
\mathtt{Passive}_1 &\equiv 1 : \surd.\mathtt{Passive}_2 \\
\mathtt{Passive}_2 &\equiv \omega : \mathtt{wakeup}.\mathtt{Active}_1 + 1 : \surd.\mathtt{Passive}_1 \\
\\
\mathtt{Activity}_n(i) &\equiv \prod^i \mathtt{Active}_1 \times \prod^{n-i} \mathtt{Passive}_1
\end{aligned}
$$

$$\begin{tikzcd}[column sep=large, row sep=large] \mathtt{Active}_1 \arrow[d, "q"'] \arrow[dr, "p"'] \arrow[drr, "r"] & & \\ \mathtt{Active}_2 & \mathtt{Passive}_2 & \mathtt{Recruit}_2 \end{tikzcd}$$



$\omega : \overline{wakeup}$
$\mathtt{Active}_1$
q
p
r
$\mathtt{Active}_2$
$\mathtt{Passive}_2$
$\mathtt{Recruit}_2$
$\mathtt{Passive}_1$


$\omega : wakeup$
$\omega : \overline{wakeup}$
$\texttt{Active}_1$
q
p
r
$\texttt{Active}_2$
$\texttt{Passive}_2$
$\texttt{Recruit}_2$
$\texttt{Passive}_1$

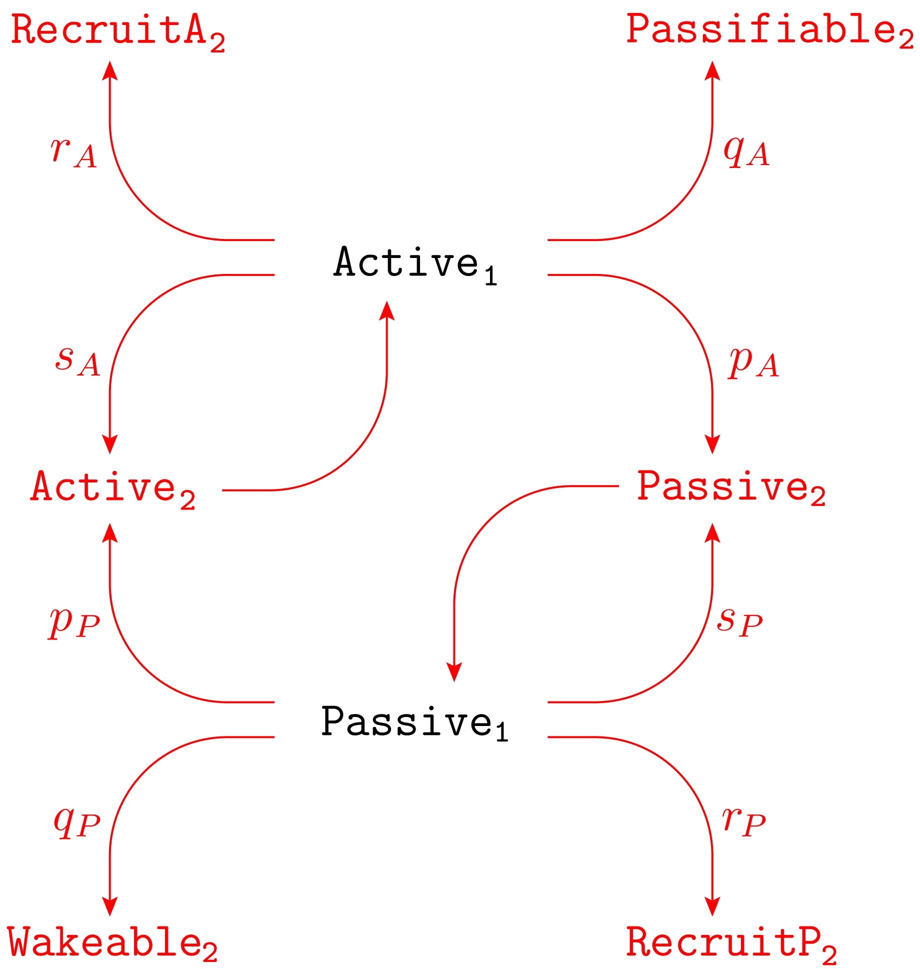
Activity Extinction

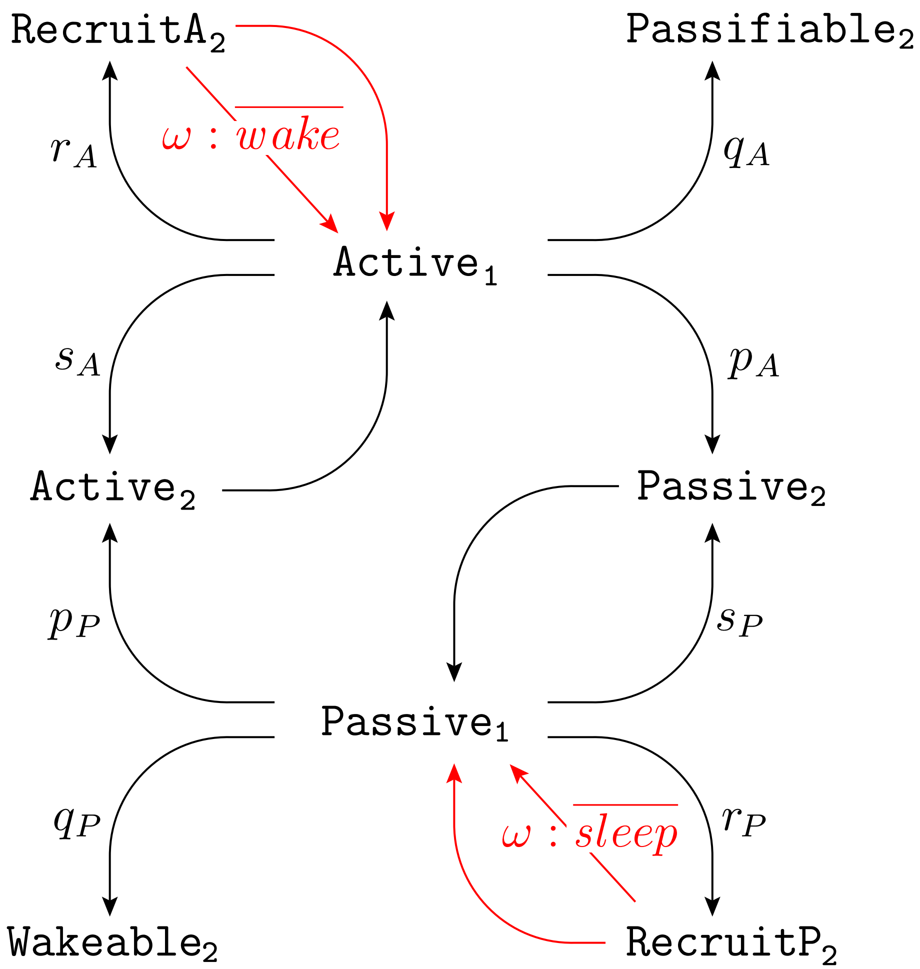
- Activity extinction occurs when number of active ants reaches 0
- Traditional Markov chain analysis tells us system will become extinct eventually
- But an entire colony can be woken up from a single active ant: intruder alarm

Ant Agent Model (II)

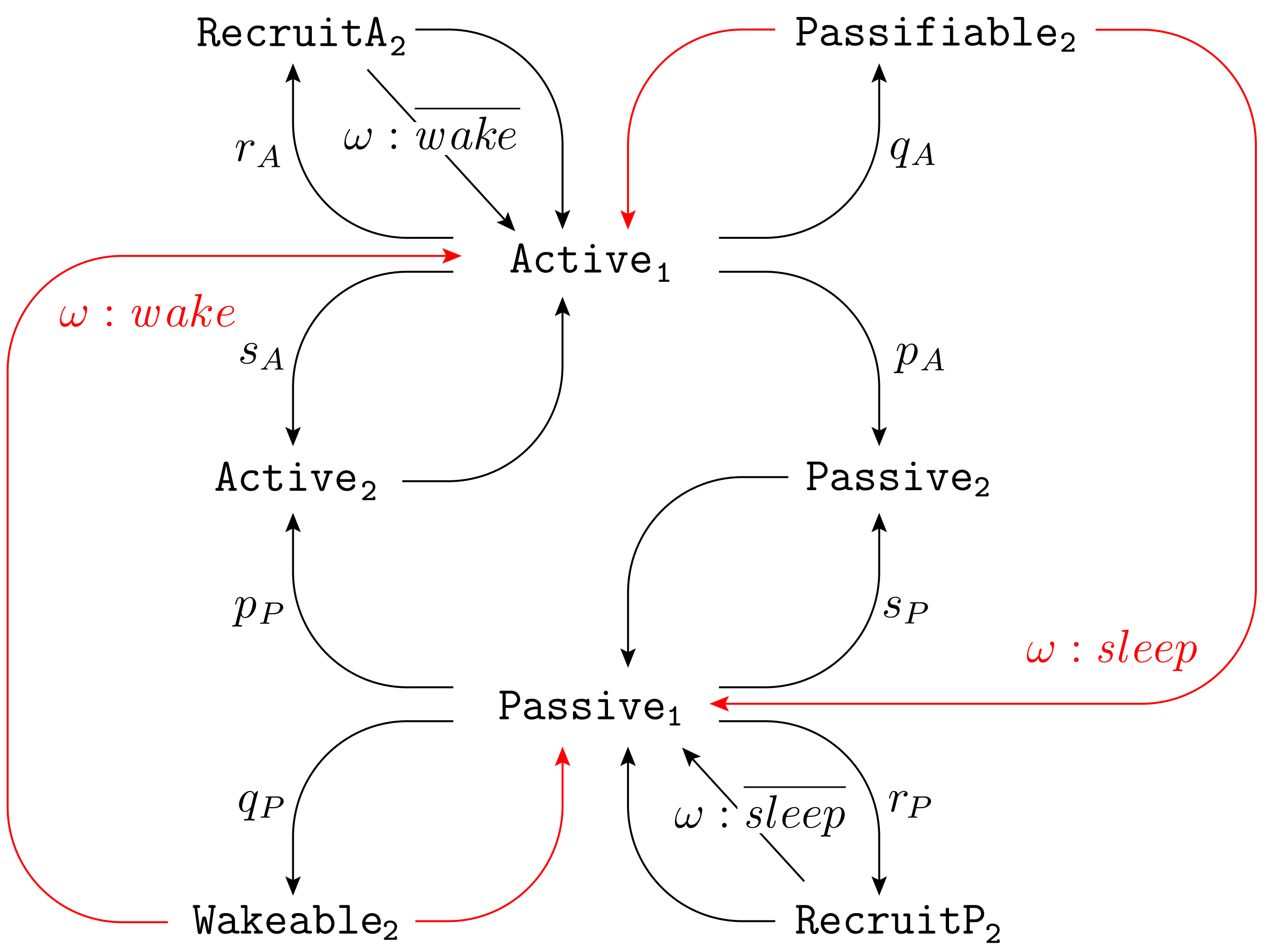
- As with previous model but with two types of recruiter ant:

  1. Allows passive ants to send active ants to sleep
  2. Allows active ants to wake passive ants up


RecruitA$_2$
Passifiable$_2$
r_A
q_A
Active$_1$
s_A
p_A
Active$_2$
Passive$_2$
p_P
s_P
Passive$_1$
q_P
r_P
Wakeable$_2$
RecruitP$_2$



RecruitA$_2$
Passifiable$_2$
r_A
$\omega : \overline{wake}$
q_A
Active$_1$
s_A
p_A
Active$_2$
Passive$_2$
p_P
s_P
Passive$_1$
q_P
$\omega : \overline{sleep}$
r_P
Wakeable$_2$
RecruitP$_2$


RecruitA$_2$
Passifiable$_2$
r_A
$\omega : \overline{wake}$
q_A
Active$_1$
$\omega : wake$
s_A
p_A
Active$_2$
Passive$_2$
p_P
s_P
$\omega : sleep$
Passive$_1$
q_P
$\omega : \overline{sleep}$
r_P
Wakeable$_2$
RecruitP$_2$

Introduction to Mean Difference Analysis

- Constructs function:

$$f(i) = \mathbb{E}(A_{t+1} | A_t = i)$$

- i.e. average activity at time $(t + 1)$ given that activity at time t is i

- Iterate equation to obtain an approximation of the activity at time t:

$$i_{t+1} = f(i_t)$$

Back to System 1

$$\begin{aligned} \texttt{Active} &\equiv p : \surd.\texttt{Passive} + q : \surd.\texttt{Active} \\ \texttt{Passive} &\equiv 1 : \surd.\texttt{Passive} \\ \\ \texttt{Colony}_n(i) &\equiv \prod^{i} \texttt{Active} \times \prod^{n-i} \texttt{Passive} \end{aligned}$$

MDE Analysis of System 1

- Let X_k represent number of active ants that the kth active ant contributes to the next time step — $\mathbb{P}(X_k = 0) = p$, and $\mathbb{P}(X_k = 1) = q$

$$A_{t+1} = X_1 + X_2 + \cdots + X_{A_t}$$

- So A_t is a binomial distribution (c.f. tossing a coin)

$$\mathbb{P}(A_{t+1} = j | A_t = i) = \left(\begin{array}{c} i \\ j \end{array} \right) q^j p^{i-j}$$

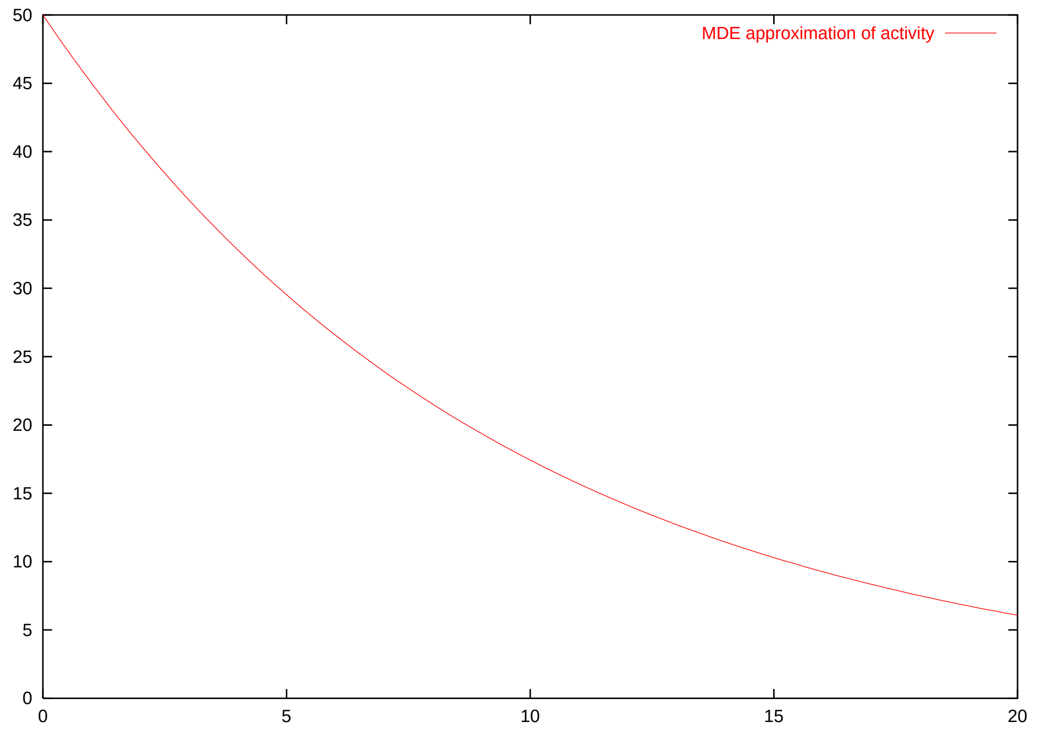
- Construct Mean Difference Equation:

$$\begin{aligned} f(i) &= \mathbb{E}(A_{t+1} | A_t = i) = qi \\ i_{t+1} &= qi_t \\ i_t &= q^t n \end{aligned}$$

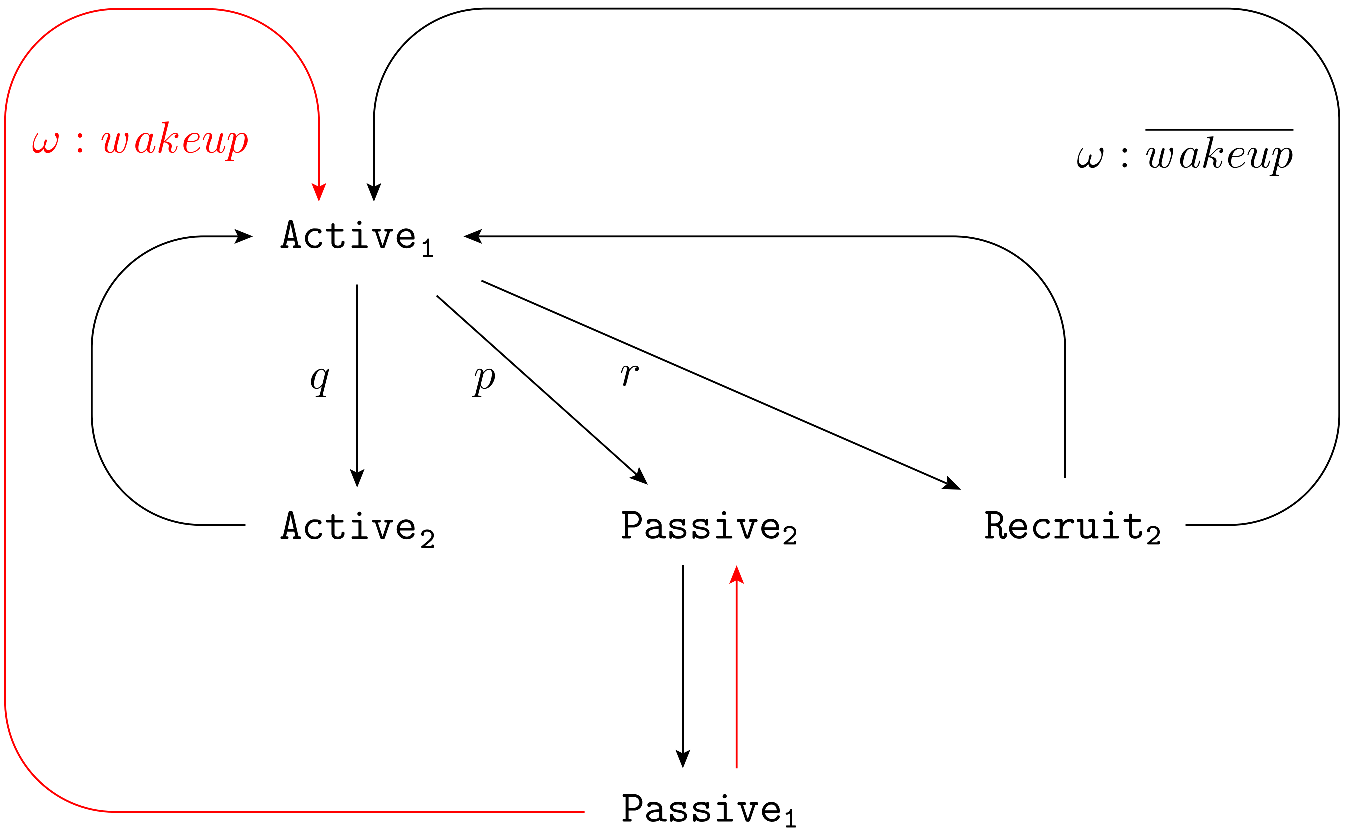
- So average activity at time t is $q^t n$ — which looks like. . .

Plot of Mean Difference Equation

System 1: $p = 0.1$, $q = 0.9$, $n = 50$. Activity against time.



Back to System 2



WSCCS Description of System 2

$$
\begin{aligned}
\texttt{Active}_1 &\equiv p : \surd.\texttt{Passive}_2 + q : \surd.\texttt{Active}_2 + r : \surd.\texttt{Recruit}_2 \\
\texttt{Active}_2 &\equiv 1 : \surd.\texttt{Active}_1 \\
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\\
\texttt{Activity}_n(i) &\equiv \prod^{i} \texttt{Active}_1 \times \prod^{n-i} \texttt{Passive}_1
\end{aligned}
$$

MDE for System 2

- Again, count number of agents of different type with random variables:

  1. A_t number of $\texttt{Active}_1$ agents
  2. Q_t number of $\texttt{Active}_2$ agents
  3. R_t number of $\texttt{Recruit}_2$ agents

- Look at single tick event $(\surd)$ from $\texttt{Activity}_n(i)$

$$\Rightarrow \; Q_t \sim \mathrm{Bin}(i, q), \; R_t \sim \mathrm{Bin}(i, r)$$

$$\Rightarrow \left\{ \begin{array}{l} \mathbb{E}(Q_t | A_t = i) = qi \\ \mathbb{E}(R_t | A_t = i) = ri \end{array} \right.$$

- A second tick event ($\surd$) takes:

$$\prod^{a} \mathtt{Active}_2 \times \prod^{b} \mathtt{Recruit}_2 \times \prod^{n-a-b} \mathtt{Passive}_2 \longrightarrow \mathtt{Activity}_n(j)$$

- This gives the map:

$$\mathbb{E}(A_{t+1} | Q_t = a, R_t = b) = \begin{cases} a + 2b & : \text{if } b < n - a - b \\ & \quad \text{(i.e. Passives exceed Recruiters)} \\ n & : \text{if } b > n - a - b \\ & \quad \text{(i.e. Recruiters exceed Passives)} \end{cases}$$

$$= \min(a + 2b, n)$$

- Combining the two 1-tick maps into a single 2-tick map:

$$\mathbb{E}(A_{t+1}|A_t = i) = \min((q + 2r)i, n)$$

- Iterate the map:

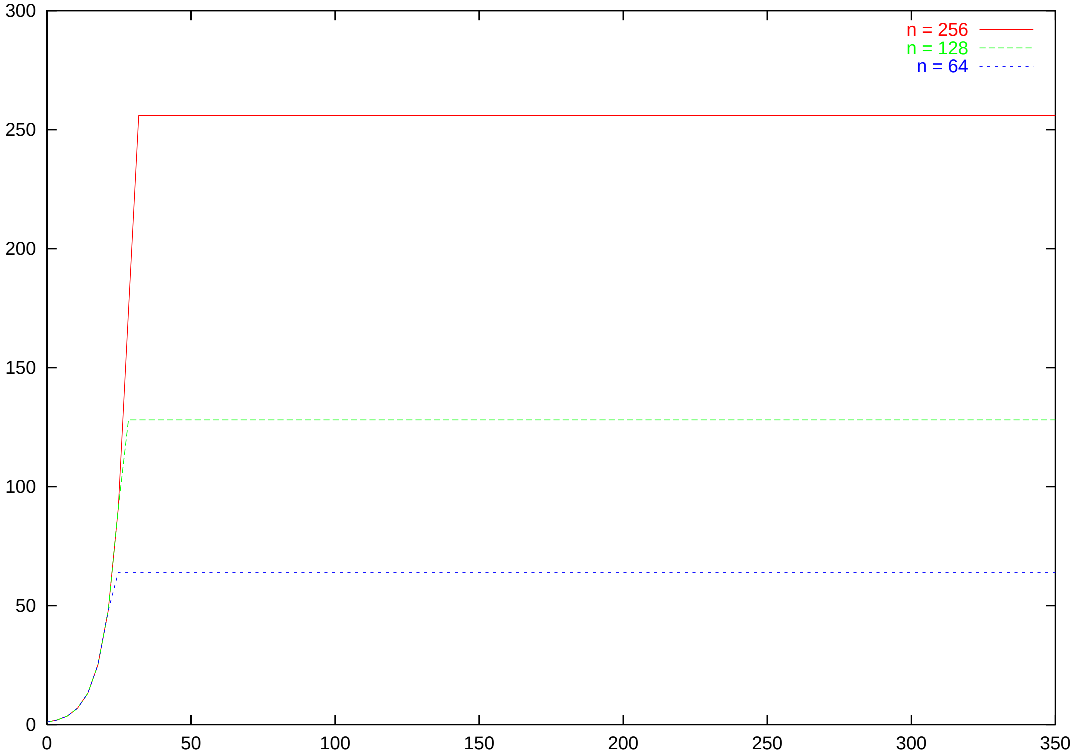
$$i_{t+1} = \min((q + 2r)i_t, n)$$

- Fixed points if: $i_t = n$ and $q + 2r > 1$ or $i_t = 0$

- Solution: $i_t = \min((q + 2r)^t i_0, n)$

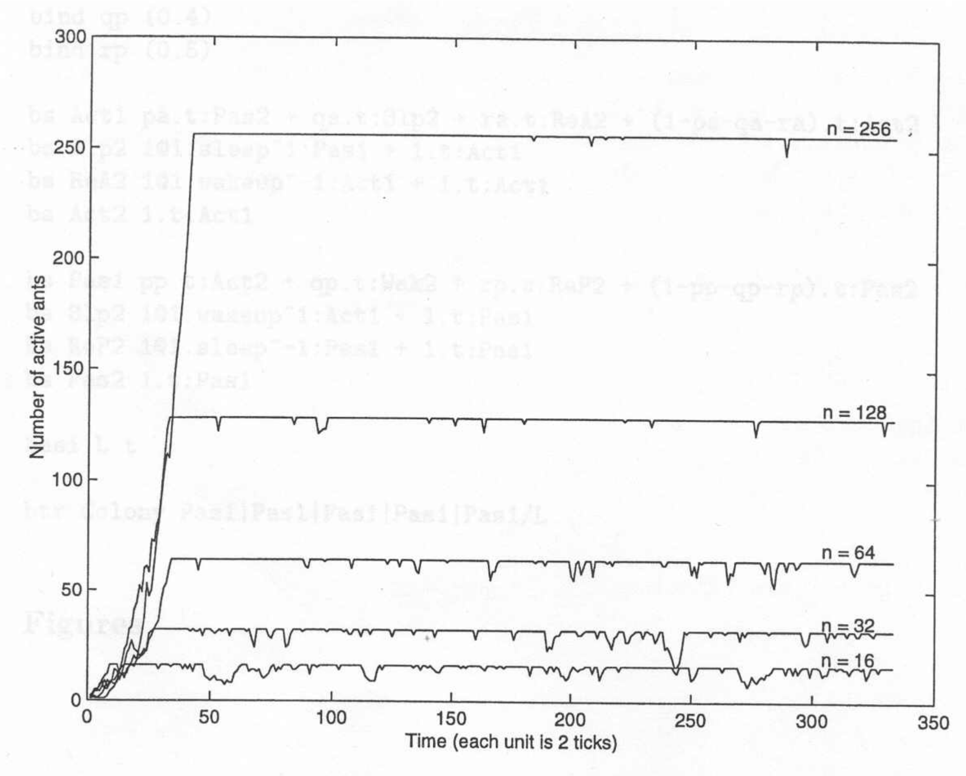
MDE: Number of Active Ants

System 2: $p = 0.2$, $q = 0.3$, $r = 0.5$



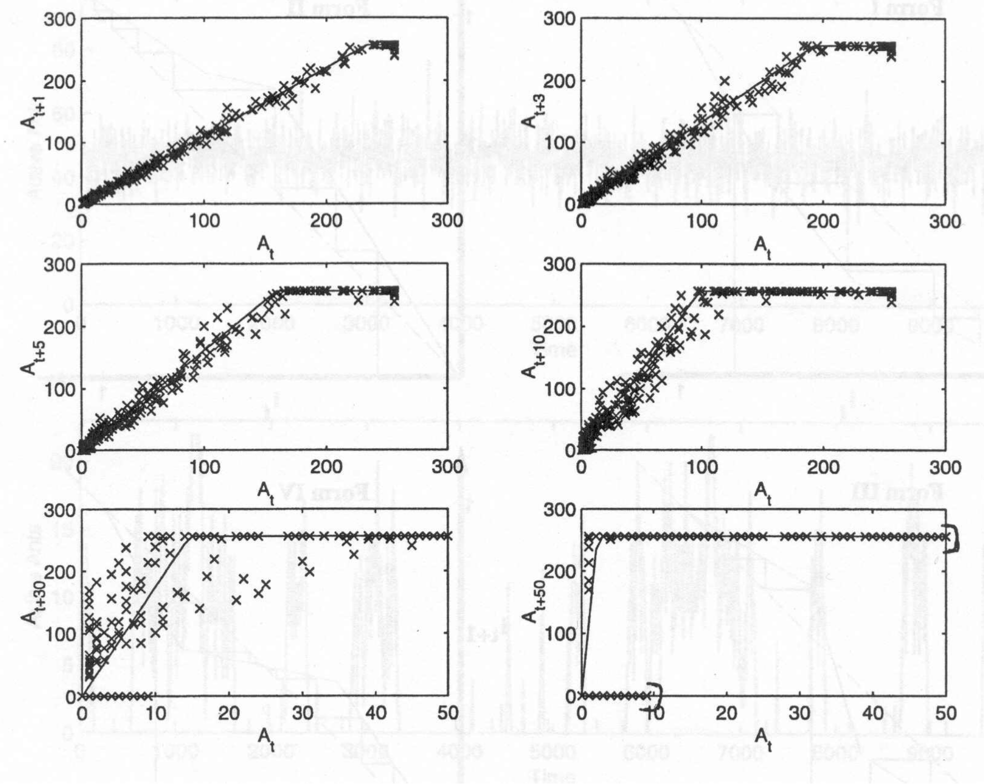
Simulations: Number of Active Ants

System 2: $p = 0.2$, $q = 0.3$, $r = 0.5$. Activity against time.



Accuracy of Mean Difference Equation

System 2: $p = 0.2$, $q = 0.3$, $r = 0.5$. A_t against A_{t+s} for $s \in \{1, 3, 5, 10, 30, 50\}$.



Mean Field Analysis and PEPA

- Currently MFA is tailored for CCS-style process algebra

  $\Rightarrow$ require different pairwise synchronisation style

- Counting components in a given state for a continuous-time process algebra will require Mean Differential Equations

- Require greater heterogeneity of models – agent classes?

Pairwise Cooperation in PEPA?

For $n = 2$:

$$\bigoplus_{i=1}^{n} {}^{<L>} P_i = P_1 \underset{L}{\bowtie} P_2$$

For $n > 2$:

$$\begin{aligned}
\bigoplus_{i=1}^{n} {}^{<L>} P_i &= P_1 \underset{L}{\bowtie} (P_2 || \cdots || P_n) + P_2 \underset{L}{\bowtie} (P_1 || P_3 || \cdots || P_n) \\
&\quad + \cdots + P_n \underset{L}{\bowtie} (P_1 || \cdots || P_{n-1}) \\
&= \sum_{j=1}^{n} P_j \underset{L}{\bowtie} \left(\prod_{k \neq j} P_k \right)
\end{aligned}$$

Conclusion

- Simple agent dynamics can produce complex non-linear global behaviour
- Mean difference equations are a good way of looking at global time-varying behaviour without looking at the global state space
- Uses process algebra descriptions as they were always meant to be used!
- Applications to the wider SPA community as a component-based transient analysis technique?