Probing the sources of suboptimality in human Bayesian inference

Luigi ACERBI [1]*, **Sethu VIJAYAKUMAR** [1], **Daniel M. WOLPERT** [2]

[1] *School of Informatics, The University of Edinburgh, UK* [2] *Department of Engineering, University of Cambridge, UK*

Introduction

Human performance in sensorimotor estimation tasks typically shows that:

- under **Gaussian** distributions of stimuli, learning is quick and biases are generally 'close-to-optimal' according to Bayesian Decision Theory (BDT) [1];
- under **complex** (e.g. skewed, bimodal) distributions, learning is slow and biases are often suboptimal [1, 2].

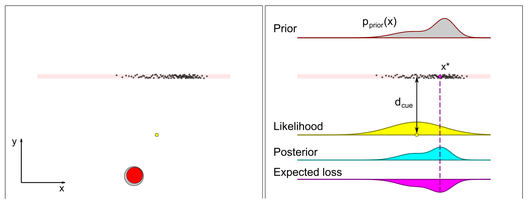
These studies do not separate whether these limitations in probabilistic inference arise from the inability to learn or inability to compute with the experimental distribution ('prior').

Here we probe people's ability at performing Bayesian inference in a **target estimation task with explicit probabilistic information**, under a variety of priors.

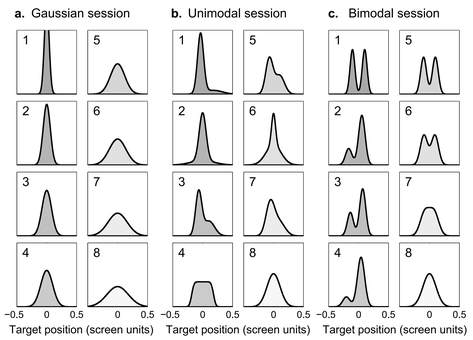
We test several hypotheses about the sources of suboptimality with a **factorial model comparison** [3].

Methods

Target estimation task:



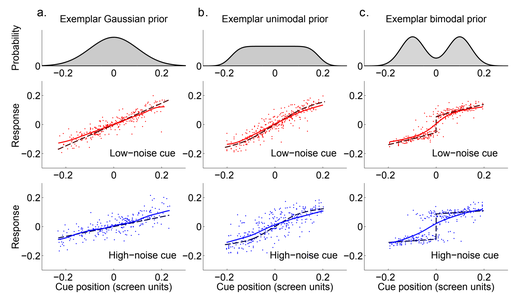
Prior distributions:



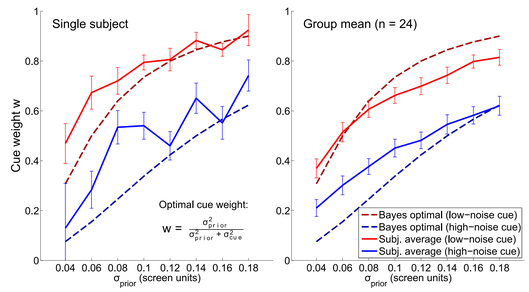
Gaussian training session followed by test session with Gaussian, unimodal or bimodal priors.

Results

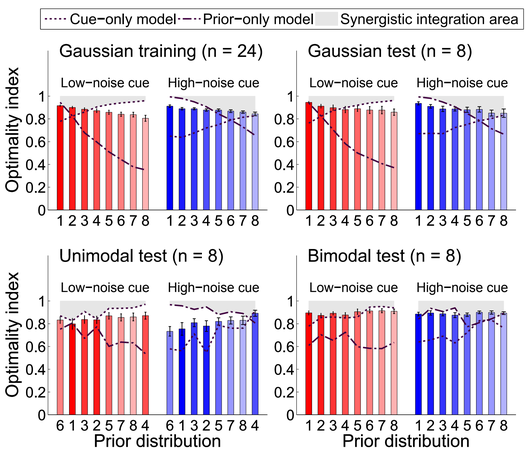
Subjects' responses:



Gaussian training session:



Subjects' performance:



Performance is suboptimal but no statistically significant difference across different conditions.

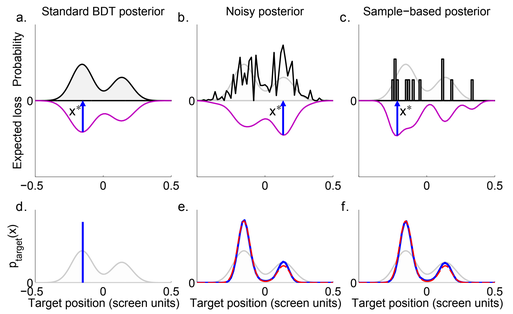
Models

Factorial model comparison:

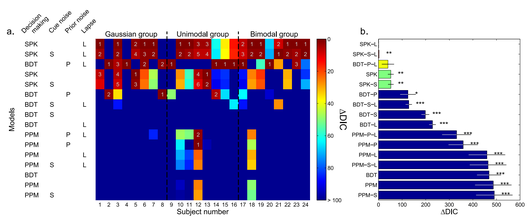
1. *Decision making:*
   - BDT: Bayesian Decision Theory;
   - PPM: posterior-probability matching; or
   - SPK: stochastic posterior, k-th power.
2. *Prior noise:* Absent or present (P).
3. *Cue-estimation noise:* Absent or present (S).
4. *Lapse:* Absent or present (L).

Stochastic posterior (SPK):

Decision noise implemented as a **stochastic representation of the posterior** (e.g. noisy posterior or sample-based approximation).
The target choise distribution $p_{target}(x)$ is well-approximated by a k-th power of the posterior.

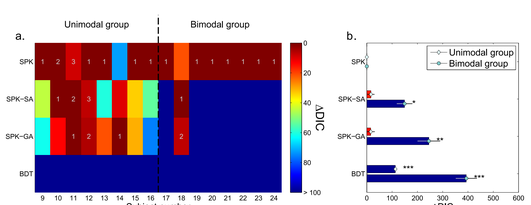


Results of model comparison:



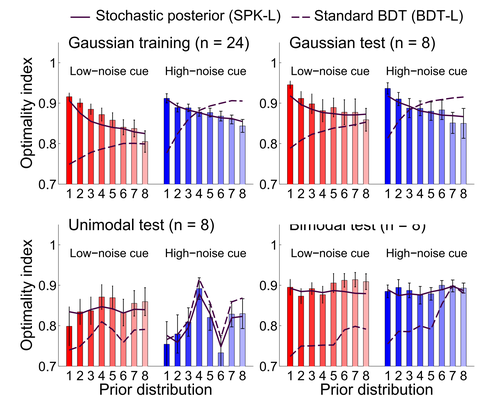
Best models **stochastic posterior with lapse** (SPK-L) and **noisy prior with lapse** (BDT-P-L).

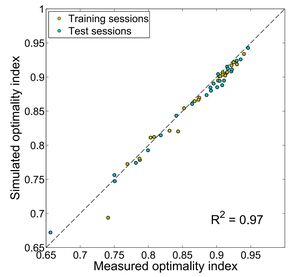
Alternatives to stochastic posterior (SPK):



Best model fit

Subjects' performance:





Summary

- Performance is suboptimal but similar across different conditions and priors.
- Decision noise distribution well-approximated by a **power of the posterior**.
- *Major sources of suboptimality:* mismatching reliability of the cues, stochastic decision making, noisy estimation of the priors.
- Other common models of variability (probability matching, sampling-average) are rejected.

References

[1] Körding, K. P., & Wolpert, D. M. Bayesian integration in sensorimotor learning. *Nature* (2004).

[2] Acerbi, L., Wolpert, D. M., & Vijayakumar, S. Internal representations of temporal statistics and feedback calibrate motor-sensory interval timing. *PLoS Comput. Biol* (2012).

[3] van den Berg, R., Awh, E., & Ma, W. J. Factorial comparison of working memory models. *Psychol. Rev.* (2014).

* *Contact:* L.Acerbi@sms.ed.ac.uk.

Acknowledegments: This research was supported partially by the EPSRC/BBSRC DTC studentship grant to LA. SV is supported by Microsoft Research, Royal Academy of Engineering and EU FP7 programs. DMW is supported by the Wellcome Trust, the Human Frontiers Science Program and the Royal Society.