

The Essence of Language Integrated Query

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Database programming languages

Kleisli

Buneman, Libkin, Suciu, Tannen, Wong (Penn)

Ferry

Grust, Mayr, Rittinger, Schreiber (Tübingen)

Links

Cooper, Lindley, Wadler, Yallop (Edinburgh)

SML#

Ohori, Ueno (Tohoku)

Ur/Web

Chlipala (Harvard/MIT)

LINQ for C#, VB, F#

Helsbjorg, Meijer, Syme (Microsoft Redmond & Cambridge)

Our goals:

Abstraction over values (first-order)

Abstraction over predicates (higher-order)

Composition of queries

Dynamic generation of queries

Type-safety

Goldilocks:

Exactly one query per *run*

Not too few (**failure**)

Not too many (**avalanche**)

Our restrictions:

We consider only *select-from-where* queries,
with *exists* and *union*.

We equate *bags* and *lists*.

Future work to extend to *group-by* and *sort-by*.

Part I

A first example

A database

people

name	age
"Alex"	60
"Bert"	56
"Cora"	33
"Drew"	31
"Edna"	21
"Fred"	60

couples

her	him
"Alex"	"Bert"
"Cora"	"Drew"
"Edna"	"Fred"

A query in SQL

```
select w.name as name, w.age – m.age as diff
from couples as c,
      people as w,
      people as m
where c.her = w.name and c.him = m.name and
      w.age > m.age
```

name	diff
“Alex”	4
“Cora”	2

A database as data

{people =

[{name = "Alex" ; age = 60};

{name = "Bert" ; age = 56};

{name = "Cora" ; age = 33};

{name = "Drew"; age = 31};

{name = "Edna"; age = 21};

{name = "Fred" ; age = 60}] ;

couples =

[{her = "Alex" ; him = "Bert" } ;

{her = "Cora" ; him = "Drew"} ;

{her = "Edna" ; him = "Fred" }] }

Importing the database (naive)

```
type DB =  
  {people :  
    {name : string; age : int} list;  
  couples :  
    {her : string; him : string} list}  
let db' : DB = database("People")
```

A query as a comprehension (naive)

```
let differences' : {name : string; diff : int} list =  
  for c in db'.couples do  
  for w in db'.people do  
  for m in db'.people do  
  if c.her = w.name && c.him = m.name && w.age > m.age then  
  yield {name : w.name; diff : w.age – m.age}
```

differences'

```
[ {name = "Alex"; diff = 4}  
  {name = "Cora"; diff = 2} ]
```

Importing the database (quoted)

```
type DB =  
  {people :  
    {name : string; age : int} list;  
  couples :  
    {her : string; him : string} list}  
let db : Expr< DB > = <@ database("People") @>
```

A query as a comprehension (quoted)

```
let differences : Expr< {name : string; diff : int} list > =  
  <@ for c in (%db).couples do  
    for w in (%db).people do  
      for m in (%db).people do  
        if c.her = w.name && c.him = m.name && w.age > m.age then  
          yield {name : w.name; diff : w.age – m.age} @>
```

run(differences)

```
[ {name = "Alex"; diff = 4}  
  {name = "Cora"; diff = 2} ]
```

Execute **run** as follows:

1. compute quoted expression
2. simplify quoted expression
3. translate query to SQL
4. execute SQL
5. translate answer to host language

Each **run** generates one query if:

- A. answer type is flat (bag-of-record-of-scalars)
- B. only permitted operations (e.g., no recursion)
- C. consistent use of **database** (all same)

Part II

Abstraction, composition, dynamic generation

Abstracting over values

```
type Names = {name : string} list
```

```
let range : Expr< (int, int) → Names > =
```

```
<@ fun(a, b) → for w in (%db).people do
```

```
    if a ≤ w.age && w.age < b then
```

```
        yield {name : w.name} @>
```

```
run(<@ (%range)(30, 40) @>)
```

```
[{name = "Cora"}; {name = "Drew"}]
```

Abstracting over a predicate

```
let satisfies : Expr< (int → bool) → Names > =  
  <@ fun(p) → for w in (%db).people do  
    if p(w.age) then  
      yield {name : w.name} @>
```

```
run(<@ (%satisfies)(fun(x) → 30 ≤ x && x < 40) @>)  
  [{name = "Cora"}; {name = "Drew"}]
```

```
run(<@ (%satisfies)(fun(x) → x mod 2 = 0) @>)  
  [{name = "Alex"}; {name = "Bert"}; {name = "Fred"}]
```


Composing queries

```
let getAge : Expr< string → int list > =
```

```
<@ fun(s) → for u in (%db).people do  
    if u.name = s then  
        yield u.age @>
```

```
let compose : Expr< (string, string) → Names > =
```

```
<@ fun(s, t) → for a in (%getAge)(s) do  
    for b in (%getAge)(t) do  
        (%range)(a, b) @>
```

```
run(<@ (%compose)("Edna", "Bert") @>)
```

```
[ {name = "Cora"}; {name = "Drew"}; {name = "Edna"} ]
```

Dynamically generated queries (1)

type Predicate =

| Above **of** int

| Below **of** int

| And **of** Predicate × Predicate

| Or **of** Predicate × Predicate

| Not **of** Predicate

let t₀ : Predicate = And(Above(30), Below(40))

let t₁ : Predicate = Not(Or(Below(30), Above(40)))

Dynamically generated queries (2)

let rec P(*t* : Predicate) : Expr<int → bool> =

match *t* **with**

| Above(*a*) → <@ **fun**(*x*) → (%**lift**(*a*)) ≤ *x* @>

| Below(*a*) → <@ **fun**(*x*) → *x* < (%**lift**(*a*)) @>

| And(*t*, *u*) → <@ **fun**(*x*) → (%P(*t*))(*x*) && (%P(*u*))(*x*) @>

| Or(*t*, *u*) → <@ **fun**(*x*) → (%P(*t*))(*x*) || (%P(*u*))(*x*) @>

| Not(*t*) → <@ **fun**(*x*) → **not**((%P(*t*))(*x*)) @>

Dynamically generated queries (3)

$P(t_0)$

$\langle @ \text{fun}(x) \rightarrow (\text{fun}(x) \rightarrow 30 \leq x)(x) \ \&\& \ (\text{fun}(x) \rightarrow x < 40)(x) \ @ \rangle$

$\langle @ \text{fun}(x) \rightarrow 30 \leq x \ \&\& \ x < 40 \ @ \rangle$

run($\langle @ (\%satisfies)(\%P(t_0)) \ @ \rangle$)

[{name = "Cora"}; {name = "Drew"}]

run($\langle @ (\%satisfies)(\%P(t_1)) \ @ \rangle$)

[{name = "Cora"}; {name = "Drew"}]

Part III

Nesting

Flat data

```
{departments =
```

```
  [ {dpt = "Product"};
```

```
    {dpt = "Quality"};
```

```
    {dpt = "Research"};
```

```
    {dpt = "Sales"} ];
```

```
employees =
```

```
  [ {dpt = "Product"; emp = "Alex"};
```

```
    {dpt = "Product"; emp = "Bert"};
```

```
    {dpt = "Research"; emp = "Cora"};
```

```
    {dpt = "Research"; emp = "Drew"};
```

```
    {dpt = "Research"; emp = "Edna"};
```

```
    {dpt = "Sales"; emp = "Fred"} ];
```

Flat data (continued)

tasks =

```
[ {emp = "Alex"; tsk = "build"};  
  {emp = "Bert"; tsk = "build"};  
  {emp = "Cora"; tsk = "abstract"};  
  {emp = "Cora"; tsk = "build"};  
  {emp = "Cora"; tsk = "design"};  
  {emp = "Drew"; tsk = "abstract"};  
  {emp = "Drew"; tsk = "design"};  
  {emp = "Edna"; tsk = "abstract"};  
  {emp = "Edna"; tsk = "call"};  
  {emp = "Edna"; tsk = "design"};  
  {emp = "Fred"; tsk = "call"} ] }
```

Importing the database

```
type Org = {departments : {dpt : string} list;  
            employees : {dpt : string; emp : string} list;  
            tasks : {emp : string; tsk : string} list }  
let org : Expr< Org > = <@ database("Org") @>
```


Departments where every employee can do a given task

```
let expertise' : Expr< string → {dpt : string} list > =
```

```
<@ fun(u) → for d in (%org).departments do
```

```
  if not(exists(
```

```
    for e in (%org).employees do
```

```
    if d.dpt = e.dpt && not(exists(
```

```
      for t in (%org).tasks do
```

```
        if e.emp = t.emp && t.tsk = u then yield { })
```

```
    )) then yield { })
```

```
  )) then yield {dpt = d.dpt} @>
```

```
run(<@ (%expertise')("abstract") @>)
```

```
[ {dpt = "Quality"}; {dpt = "Research"} ]
```

Nested data

```
[ {dpt = "Product"; employees =  
  [ {emp = "Alex"; tasks = ["build"] }  
    {emp = "Bert"; tasks = ["build"] } ] ];  
{dpt = "Quality"; employees = [ ] };  
{dpt = "Research"; employees =  
  [ {emp = "Cora"; tasks = ["abstract"; "build"; "design"] };  
    {emp = "Drew"; tasks = ["abstract"; "design"] };  
    {emp = "Edna"; tasks = ["abstract"; "call"; "design"] } ] ];  
{dpt = "Sales"; employees =  
  [ {emp = "Fred"; tasks = ["call"] } ] ] ]
```


Higher-order queries

let any : Expr < (A list, A → bool) → bool > =

<@ **fun**(xs, p) →
 exists(for x in xs do
 if p(x) **then**
 yield { }) @>

let all : Expr < (A list, A → bool) → bool > =

<@ **fun**(xs, p) →
 not((%any)(xs, **fun**(x) → **not**(p(x)))) @>

let contains : Expr < (A list, A) → bool > =

<@ **fun**(xs, u) →
 (%any)(xs, **fun**(x) → x = u) @>

Departments where every employee can do a given task

```
let expertise : Expr< string → {dpt : string} list > =  
  <@ fun(u) → for d in (%nestedOrg)  
    if (%all)(d.employees,  
      fun(e) → (%contains)(e.tasks, u) then  
    yield {dpt = d.dpt} @>
```

```
run(<@ (%expertise)("abstract") @>)
```

```
[{dpt = "Quality"}; {dpt = "Research"}]
```

Part IV

Quotations vs. functions

Abstracting over values

```
let range : Expr< (int, int) → Names > =  
  <@ fun(a, b) → for w in (%db).people do  
    if a ≤ w.age && w.age < b then  
      yield {name : w.name} @>  
run(<@ (%range)(30, 40) @>)
```

vs.

```
let range'(a : Expr< int >, b : Expr< int >) : Names =  
  <@ for w in (%db).people do  
    if (%a) ≤ w.age && w.age < (%b) then  
      yield {name : w.name} @>  
run(range'(<@ 30 @>, <@ 40 @>))
```

Composing queries

let compose : Expr< (**string**, **string**) → Names > =
 <@ **fun**(s, t) → **for a in** (%getAge)(s) **do**
 for b in (%getAge)(t) **do**
 (%range)(a, b) @>

vs.

let compose' : Expr< (**string**, **string**) → Names > =
 <@ **fun**(s, t) → **for a in** (%getAge)(s) **do**
 for b in (%getAge)(t) **do**
 (%range'(<@ a @>, <@ b @>)) @>

Prefer
closed quotations
to
open quotations.

Prefer
quotations of functions
to
functions of quotations.

Part V

From XPath to SQL

Part VI

Idealised LINQ

Terms

VAR

$$\frac{}{\Gamma, x : A \vdash x : A}$$

FUN

$$\frac{\Gamma, x : A \vdash N : B}{\Gamma \vdash \mathbf{fun}(x) \rightarrow N : A \rightarrow B}$$

APP

$$\frac{\Gamma \vdash L : A \rightarrow B \quad \Gamma \vdash M : A}{\Gamma \vdash L M : B}$$

SINGLETON

$$\frac{\Gamma \vdash M : A}{\Gamma \vdash \mathbf{yield} M : A \mathbf{list}}$$

FOR

$$\frac{\Gamma \vdash M : A \mathbf{list} \quad \Gamma, x : A \vdash N : B \mathbf{list}}{\Gamma \vdash \mathbf{for} x \mathbf{in} M \mathbf{do} N : B \mathbf{list}}$$

REC

$$\frac{\Gamma, f : A \rightarrow B, x : A \vdash N : B}{\Gamma \vdash \mathbf{rec} f(x) \rightarrow N : A \rightarrow B}$$

Quoted terms

VARQ

$$\frac{}{\Gamma; \Delta, x : A \vdash x : A}$$

FUNQ

$$\frac{\Gamma; \Delta, x : A \vdash N : B}{\Gamma; \Delta \vdash \mathbf{fun}(x) \rightarrow N : A \rightarrow B}$$

APPQ

$$\frac{\Gamma; \Delta \vdash L : A \rightarrow B \quad \Gamma; \Delta \vdash M : A}{\Gamma; \Delta \vdash L M : B}$$

SINGLETONQ

$$\frac{\Gamma; \Delta \vdash M : A}{\Gamma; \Delta \vdash \mathbf{yield} M : A \mathbf{list}}$$

FORQ

$$\frac{\Gamma; \Delta \vdash M : A \mathbf{list} \quad \Gamma; \Delta, x : A \vdash N : B \mathbf{list}}{\Gamma; \Delta \vdash \mathbf{for} x \mathbf{in} M \mathbf{do} N : B \mathbf{list}}$$

DATABASE

$$\frac{\Sigma(\mathbf{db}) = \{\overline{\ell : T}\}}{\Gamma; \Delta \vdash \mathbf{database}(\mathbf{db}) : \{\overline{\ell : T}\}}$$

Quotation and anti-quotation

QUOTE

$$\frac{\Gamma; \cdot \vdash M : A}{\Gamma \vdash \langle @ M @ \rangle : \text{Expr} \langle A \rangle}$$

ANTIQUOTE

$$\frac{\Gamma \vdash M : \text{Expr} \langle A \rangle}{\Gamma; \Delta \vdash (\%M) : A}$$

RUN

$$\frac{\Gamma \vdash M : \text{Expr} \langle T \rangle}{\Gamma \vdash \mathbf{run}(M) : T}$$

LIFT

$$\frac{\Gamma \vdash M : O}{\Gamma \vdash \mathbf{lift}(M) : \text{Expr} \langle O \rangle}$$

Normalisation: symbolic evaluation

(fun(x) $\rightarrow N$) $M \rightsquigarrow N[x := M]$

$\{\overline{\ell = M}\}.l_i \rightsquigarrow M_i$

for x **in** (yield M) **do** $N \rightsquigarrow N[x := M]$

for y **in** (for x **in** L **do** M) **do** $N \rightsquigarrow$ **for** x **in** L **do** (for y **in** M **do** N)

for x **in** (if L **then** M) **do** $N \rightsquigarrow$ **if** L **then** (for x **in** M **do** N)

for x **in** [] **do** $N \rightsquigarrow$ []

for x **in** (L @ M) **do** $N \rightsquigarrow$ (for x **in** L **do** N) @ (for x **in** M **do** N)

if true **then** $M \rightsquigarrow M$

if false **then** $M \rightsquigarrow$ []

Normalisation: *ad hoc* rewriting

for x **in** L **do** $(M @ N)$ \hookrightarrow (**for** x **in** L **do** M) @ (**for** x **in** L **do** N)

for x **in** L **do** $[\]$ \hookrightarrow $[\]$

if L **then** $(M @ N)$ \hookrightarrow (**if** L **then** M) @ (**if** L **then** N)

if L **then** $[\]$ \hookrightarrow $[\]$

if L **then** (**for** x **in** M **do** N) \hookrightarrow **for** x **in** M **do** (**if** L **then** N)

if L **then** (**if** M **then** N) \hookrightarrow **if** ($L \ \&\& \ M$) **then** N

yield x \hookrightarrow **yield** $\{\overline{\ell = x.\ell}\}$

database(db). ℓ \hookrightarrow **for** x **in** **database**(db). ℓ **do** **yield** x

Properties of reduction

On well-typed terms, the relations \rightsquigarrow and \hookrightarrow

- *preserve* typing,
- are *strongly normalising*, and
- are *confluent*.

Terms in normal form under \rightsquigarrow satisfy the *subformula property*: with the exception of predicates (such as $<$ or **exists**), the type of a subterm must be a subformula of either the type of a free variable or of the type of the term.

Example (1): query

```
run(<@ (%compose)("Edna", "Bert") @>)
```

Example (2): after splicing

```
(fun(s, t) →  
  for a in (fun(s) →  
    for u in database("People").people do  
    if u.name = s then yield u.age)(s) do  
  for b in (fun(s) →  
    for u in database("People").people do  
    if u.name = s then yield u.age)(t) do  
  (fun(a, b) →  
    for w in database("People").people do  
    if a ≤ w.age && w.age < b then  
    yield {name : w.name})(a, b))  
("Edna", "Bert")
```

Example (3): beta reduction \rightsquigarrow

```
for a in (for u in database("People").people do  
    if u.name = "Edna" then yield u.age) do  
for b in (for u in database("People").people do  
    if u.name = "Bert" then yield u.age) do  
for w in database("People").people do  
if a ≤ w.age && w.age < b then  
yield {name : w.name}
```

Example (4): other rewriting \rightsquigarrow

```
for u in database("People").people do  
if u.name = "Edna" then  
for v in database("People").people do  
if v.name = "Bert" then  
for w in database("People").people do  
if u.age  $\leq$  w.age && w.age < v.age then  
yield {name : w.name}
```

Example (5): *ad hoc* reductions \hookrightarrow

```
for u in database("People").people do  
for v in database("People").people do  
for w in database("People").people do  
if u.name = "Edna" && v.name = "Bert" &&  
    u.age  $\leq$  w.age && w.age < v.age then  
yield {name : w.name}
```

Example (6): SQL

```
select w.name as name
from people as u,
      people as v,
      people as w
where u.name = "Edna" and v.name = "Bert" and
      u.age ≤ w.age and w.age < v.age
```

Part VII

Results

Example	F# 2.0	F# 3.0	ILINQ	<i>norm</i>
differences	17.6	20.6	18.1	0.5
range	×	5.6	2.9	0.3
satisfies	2.6	×	2.9	0.3
satisfies	4.4	×	4.6	0.3
compose	×	×	4.0	0.8
P(t ₀)	2.8	×	3.3	0.3
P(t ₁)	2.7	×	3.0	0.3
expertise'	7.2	9.2	8.0	0.6
expertise	×	66.7 ^{av}	8.3	0.9
xp ₀	×	8.3	7.9	1.9
xp ₁	×	14.7	13.4	1.1
xp ₂	×	17.9	20.7	2.2
xp ₃	×	3744.9	3768.6	4.4

All times in milliseconds. ^{av} marks query avalanche.

Our goals:

Abstraction over values (first-order)

Abstraction over predicates (higher-order)

Composition of queries

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Not too many (**avalanche**)



Daddy
as a
lambada
man

Appendix A7

Problems with F#

Problems with F# PowerPack

(Notes from James Cheney)

Problems fixed in F# PowerPack code:

- F# 2.0/PowerPack lacked support for singletons in nonstandard places (i.e. other than in a comprehension body).
- F# 2.0/PowerPack also lacked support for Seq.exists in certain places because it was assuming that expressions of base types (eg. booleans) did not need to be further translated.

F# 3.0:

- Did not exhibit the above problems
- But did exhibit translation bug where something like

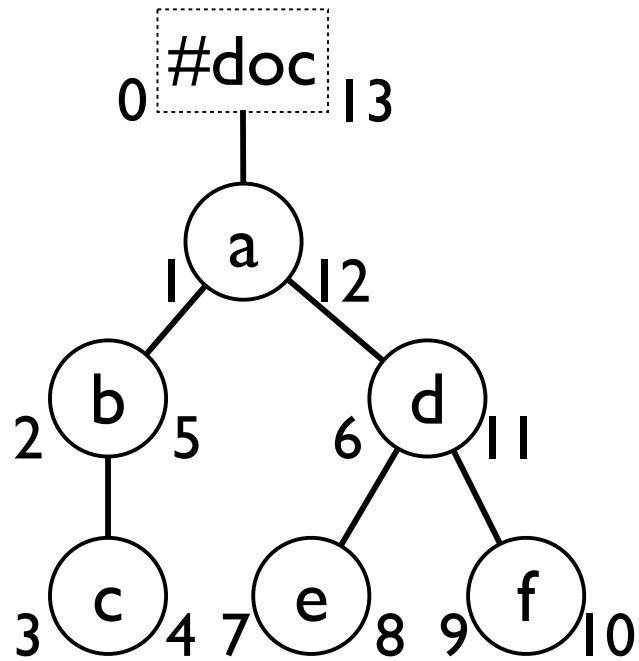
```
query if 1 = 2 then yield 3
```

leads to a run-time type error.

Appendix A7

From XPath to SQL

Representing XML



xml

id	parent	name	pre	post
0	-1	#doc	0	13
1	0	a	1	12
2	1	b	2	5
3	2	c	3	4
4	1	d	6	11
5	4	e	7	8
6	4	f	9	10

type Node =

{id : **int**, parent : **int**, name : **string**, pre : **int**, post : **int**}

Abstract syntax of XPath

type Axis =

- | Self
- | Child
- | Descendant
- | DescendantOrSelf
- | Following
- | FollowingSibling
- | Rev **of** Axis

type Path =

- | Seq **of** Path × Path
- | Axis **of** Axis
- | NameTest **of** string
- | Filter **of** Path

An evaluator for XPath: axis

let rec axis(ax : Axis) : Expr< (Node, Node) → **bool** > =

match ax **with**

| Self → <@ **fun**(s, t) → s.id = t.id @>

| Child → <@ **fun**(s, t) → s.id = t.parent @>

| Descendant → <@ **fun**(s, t) →
s.pre < t.pre && t.post < s.post @>

| DescendantOrSelf → <@ **fun**(s, t) →
s.pre ≤ t.pre && t.post ≤ s.post @>

| Following → <@ **fun**(s, t) → s.pre < t.pre @>

| FollowingSibling → <@ **fun**(s, t) →
s.post < t.pre && s.parent = t.parent @>

| Rev(axis) → <@ **fun**(s, t) → (%axis(ax))(t, s) @>

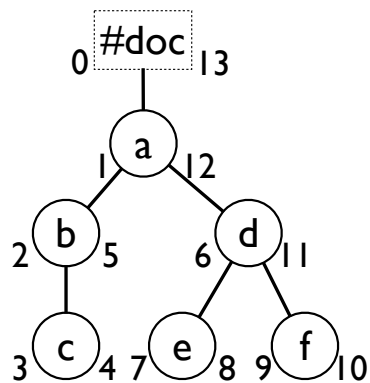
An evaluator for XPath: path

```
let rec path(p : Path) : Expr< (Node, Node) → bool > =  
match p with  
| Seq(p, q) → <@ fun(s, u) → (%any)((%db).xml,  
    fun(t) → (%path(p))(s, t) && (%path(q))(t, u)) @>  
| Axis(ax) → axis(ax)  
| NameTest(name) → <@ fun(s, t) →  
    s.id = t.id && s.name = name @>  
| Filter(p) → <@ fun(s, t) → s.id = t.id &&  
    (%any)((%db).xml, fun(u) → (%path(p))(s, u)) @>
```

An evaluator for XPath: xpath

```
let xpath(p : Path) : Expr< Node list > =  
  <@ for root in (%db).xml do  
    for s in (%db).xml do  
      if root.parent = -1 && (%path(p))(root, s) then  
        yield s @>
```

Examples



`/*/*`

`run(xpath(Seq(Axis(Child), Axis(Child))))`

`[2; 4]`

`//* [following-sibling::d]`

`run(xpath(Seq(Axis(Descendant),`

`Filter(Seq(Axis(FollowingSibling),`

`NameTest("d"))))))`

`[2]`