Query Translation from XPath to SQL in the Presence of Recursive DTDs [*]

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Abstract

We study the problem of evaluating XPath queries over XML data that is stored in an RDBMS via schema-based shredding. The interaction between recursion (descendants-axis) in XPath queries and recursion in DTDs makes it challenging to answer XPath queries using RDBMS. We present a new approach to translating XPath queries into SQL queries based on a notion of *extended* XP*ath expressions* and a simple least fixpoint (LFP) operator. Extended XPath expressions are a mild extension of XPath, and the LFP operator takes a single input relation and is already supported by most commercial RDBMS. We show that extended XPath expressions are capable of capturing both DTD recursion and XPath queries in a uniform framework. Furthermore, they can be translated into an equivalent sequence of SQL queries with the LFP operator. We present algorithms for rewriting XPath queries over a (possibly recursive) DTD into extended XPath expressions and for translating extended XPath expressions to SQL queries, as well as optimization techniques. The novelty of our approach consists in its capability to answer a large class of XPath queries by means of only low-end RDBMS features already available in most RDBMS, as well as its flexibility to accommodate existing relational query optimization techniques. In addition, these translation algorithms provide a solution to query answering for certain (possibly recursive) XML views of XML data. Our experimental results verify the effectiveness of our techniques.

1 Introduction

It is increasingly common to find XML data stored in a relational database system (RDBMS), typically based on DTD/schema-based shredding into relations [43] as found in many commercial products (e.g., [23, 35, 38]). With this comes the need for answering XML queries using RDBMS, by translating XML queries to SQL.

The query translation problem can be stated as follows. Consider a mapping τ_d, defined in terms of DTD-based shredding, from XML documents conforming to a DTD D to relations of a schema $\mathcal{R}$. Given an XML query Q, we want to find (a sequence of) *equivalent* SQL queries Q' such that for any XML document T conforming to D, Q over T can be answered by Q' over the

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relation instance $\tau_d(T)$ of $\mathcal{R}$, i.e., $Q(T) = Q'(\tau_d(T))$. Here we allow DTDs D to be recursive and consider queries Q in XPath [10], which is the core of XML query languages XQuery and XSLT.

As observed in [28], a closely related issue concerns query answering for XML views of XML data. Consider an XML view V of an XML document T. For practical reasons, e.g., XML access control [14] and data integration [31], the view V may necessarily be virtual and specified by a recursive DTD. To answer XPath queries Q posed on V without materializing V, one needs to rewrite Q into an equivalent XML query Q' on the underlying source T such that $Q(V) = Q'(T)$.

The query translation problem is, however, nontrivial: DTDs (or XML Schema) found in practice are often recursive [8] and complex. This is particularly evident in real-life applications (see, e.g., BIOML [7] and GedML [20], which, when represented as graphs, contains a number of nested and overlapping cycles). The interaction between recursion in a DTD and recursion in an XML query complicates the translation. When the DTD has a tree or DAG structure, a natural approach [24] is based on enumerating all matching paths of the input XPath query in a DTD, sharing a single representation of common sub-paths, rewriting these paths into SQL queries, and taking a union of these queries. However, this approach no longer works on recursive DTDs since it may lead to infinitely many paths when dealing with the descendant-or-self axis '//' in XPath. Another approach is by means of a rich intermediate language and middleware as proposed in [41]: first express input XML queries in the intermediate language, and then evaluate the translated queries leveraging the computing power of the middleware and the underlying RDBMS. However, as pointed out by a recent survey [28], this approach requires implementation of the middleware on top of RDBMS, and introduces communication overhead between the middleware and the RDBMS, among other things. It is more convenient and possibly more efficient to translate XPath queries to SQL and push the work (SQL queries) to the underlying RDBMS, capitalizing on the RDBMS to evaluate and optimize the queries. This, however, calls for an extension of SQL to support certain recursive operator. As observed by [28], although there has been a host of work on storing and querying XML using an RDBMS [11, 19, 24, 27, 33, 41, 42], the problem of translating recursive XML queries into SQL in the presence of recursive DTDs has not been well studied, and it was singled out as the most important open problem in [28].

Recently an elegant approach was proposed in [27] to translating path queries to SQL with the linear-recursion construct *with...recursive* of SQL'99. The algorithm of [27] is capable of translating path queries with // and limited qualifiers to (a sequence of) SQL queries with the SQL'99 recursion operator. Unfortunately, this approach has several limitations. The first weakness is that it relies on the SQL'99 recursion functionality, which is not currently supported by many commercial products including Oracle and Microsoft SQL server. One wants an effective query translation approach that works with a wide variety of products supporting low-end recursion functionality, rather than requiring an advanced DBMS feature of only the most sophisticated systems. Second, the SQL queries with the SQL'99 recursion produced by the translation algorithm of [27] are typically large and complex, and cannot be effectively optimized by all platforms supporting SQL'99 recursion for the same reasons that not all RDBMS can effectively optimize mildly complex non-recursive queries [18]. Worse still, as the *with...recursive* operator is treated as a blackbox, the user can do little to optimize it. A third problem is that the class of path query handled by the algorithm of [27] is too restricted to express XPath queries commonly found in practice. Finally, this approach does not help XPath query answering for XML views despite its analogy with XPath query translation to relational views.

In light of this we propose a new approach to translating a class of XPath queries to SQL, which also provides a solution to query answering for certain XML views of XML data. The approach is based on a notion of *extended* XPath expressions and a simple least fixpoint (LFP) operator. Extended XPath expressions generalize XPath and regular XPath [34] by supporting variables and general Kleene closure E^* instead of //. The LFP operator $\Phi(R)$ takes a single input relation R instead of multiple relations as required by the SQL'99 *with...recursion* operator. Although theoretically the *with...recursive* operator can be encoded in terms of the LFP operator, the coding introduces additional overhead. The LFP operator is already supported by many commercial systems such as Oracle (*connectby*) and IBM DB2 (*with...recursion*), and is supported by Microsoft SQL server (*common table* [37]). We show that extended XPath expressions are capable of expressing a large class of XPath queries over a (recursive) DTD D, by substituting the general Kleene closure E^* for //, and by giving a finite representation of possibly infinite matching paths of an XPath query in terms of variables and E^*, in polynomial time. That is, extended XPath expressions capture both DTD recursion and XPath recursion in a uniform and compact framework. Moreover, we show that each extended XPath expression can be rewritten to a sequence of equivalent SQL queries with the LFP operator. That is, low-end RDBMS features (SQL with $\Phi(R)$) suffice to support complex XPath queries.

Taken together, our approach works as follows. Given an XPath query Q on a (possibly recursive) DTD, we first rewrite Q into an extended XPath query E_Q that characterizes all matching paths, and then translate E_Q to an equivalent sequence Q' of SQL queries. Both E_Q and Q' are bounded by a low polynomial in the size of the input query Q and the DTD D. To this end we provide an efficient algorithm for translating an XPath query over a (recursive) DTD D to an equivalent extended XPath query, and a novel algorithm for rewriting an extended XPath query into a sequence of SQL queries with the LFP operator, as well as optimization techniques.

Contributions. The main contributions of this paper include the following.

- The notion of extended XPath expressions that captures DTD recursion and XPath recursion in a uniform framework.
- The use of the simple LFP operator commonly found in commercial products to express a large class of XPath queries.
- An efficient algorithm for rewriting XPath queries over a (possibly recursive) DTD into extended XPath queries that characterize matching paths, based on dynamic programming.
- A novel algorithm for rewriting an extended XPath expression to a sequence of SQL queries with the LFP operator.
- Experimental results verifying the effectiveness of our approach and techniques, using real-life XML DTDs.

Our approach has several salient features. (1) It requires only low-end RDBMS features instead of the advanced SQL'99 recursion functionality. As a result it provides a variety of commercial RDBMS with an immediate capability to answer XPath queries over recursive DTDs. (2) It produces SQL queries that are less complex than their counterparts generated with the

SQL'99 recursion, and can be optimized by most RDBMS platforms. Furthermore, it can easily accommodate optimization techniques developed for SQL queries, e.g., multi-query [39] and recursive SQL query optimization [40]. (3) It is capable of handling a class of XPath queries supporting child, self-or-descendants and union as well as rich qualifiers with data values, conjunction, disjunction and negation, which are beyond those studied in earlier proposals. These thus yield an effective and efficient method that works with most RDBMS products, to answer a large class of XPath queries found in practice. (4) In contrast to the approach of [27], our approach provides also an effective solution to XPath query answering for certain XML views. As recently observed in [15], the query answering problem is nontrivial because XPath is not closed under query rewriting, i.e., for an XPath query Q posed on target XML data V, there may not exist an equivalent XPath query Q' on the underlying source such that $Q(V) = Q'(T)$; worse still, even if an equivalent XPath query Q' exists and when V is specified by a nonrecursive DTD, it takes exponential time to compute Q' in the size of Q. By leveraging extended XPath, our first translation algorithm, namely, the one from XPath to extended XPath, provides an effective solution to the query answering problem for a class of XML views.

Organization. The remainder of the paper is organized as follows. Section 2 reviews DTDs, XPath and schema-based mapping from XML to relations; it also introduces extended XPath. Section 3 outlines our query translation approach as opposed to the one given in [27]. Section 4 provides an algorithm for translating XPath queries to extended XPath expressions, followed by an algorithm for rewriting extended XPath expressions into SQL with a simple LFP operator in Section 5. Experimental results are presented in Section 6, followed by related work in Section 7. Finally, Section 8 concludes the paper.

2 DTD, XPath, and Schema-Based Shredding

In this section, we review DTDs, XPath queries, and DTD-based shredding of XML data into relations. We also introduce the notion of extended XPath, an extension of a fragment of XPath.

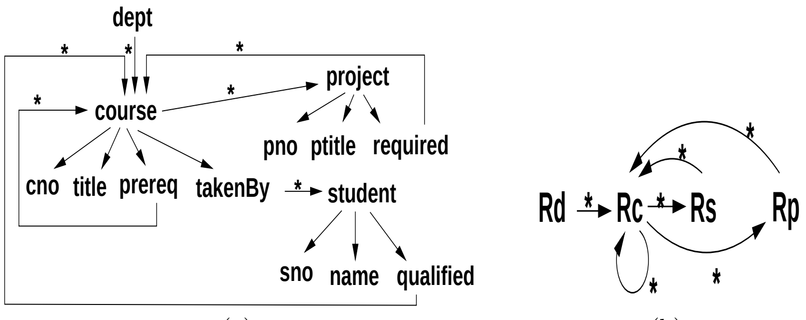
2.1 DTDs

Without loss of generality we represent a DTD D as an extended context-free grammar of the form (Ele, Rg, r), where Ele is a finite set of element types; r is a distinguished type, called the root type; and Rg defines the element types: for any A in Ele, $Rg(A)$ is a regular expression α:

$$\alpha \ ::= \quad \epsilon \quad | \quad B \quad | \quad \alpha, \alpha \quad | \quad (\alpha \mid \alpha) \quad | \quad \alpha^*,$$

where ϵ is the empty word, B is a type in Ele (referred to as a *subelement* type of A), and '|', ',' and '$*$' denote disjunction, concatenation and the Kleene star, respectively. We refer to $A \rightarrow Rg(A)$ as the *production* of A. To simplify the discussion we do not consider attributes, and we assume that an element v may possibly carry a text value (`PCDATA`) denoted by $v.val$. An XML document that conforms to a DTD is called an XML *tree of the* DTD.

Along the same lines as [43], we represent DTD D as a graph, called the DTD *graph* of D and denoted by G_D. In G_D, each node represents a distinct element type A in D, called *the A node*, and an edge represents the parent/child relationship. More specifically, for any production $A \rightarrow \alpha$, there is an edge from the A node to the B node for each subelement type B in α; the edge is labeled with '$*$' if B is enclosed in α_0^* for some sub-expression α_0 of α. This simple



(a) (b)

Figure 1: A graph representation of the `dept` DTD.

graph representation of DTDs suffices since, as will be seen shortly, we do not consider ordering in XPath. When it is clear from the context, we shall use DTD and its graph interchangeably.

A DTD D is *recursive* if it has an element type that is defined (directly or indirectly) in terms of itself. Note that the DTD graph G_D of D is *cyclic* if D is recursive. A DTD graph G_D is called a n-cycle graph if G_D consists of n simple cycles, where a simple cycle refers to a cycle in which no node appears more than once.

A DTD D is *contained* in another DTD D' if the DTD graph of D is a sub-graph of D', i.e., there is a homomorphism mapping from D to D' such that the root of D is mapped to the root of D'.

Example 2.1: We consider a `dept` DTD (E, `dept`, Rg) as our running example, where E = {`course, cno, title, prereq, takenBy, project, student, sno, name, qualified, pno, ptitle, required`}, and Rg is defined as follows:

`dept` $\rightarrow$ `course*`	`course` $\rightarrow$ `cno, title, prereq, takenBy, project*`
`prereq` $\rightarrow$ `course*`	`student` $\rightarrow$ `sno, name, qualified`
`takenBy` $\rightarrow$ `student*`	`project` $\rightarrow$ `pno, ptitle, required`
`qualified` $\rightarrow$ `course*`	`required` $\rightarrow$ `course*`

A `dept` has a list of `course` elements. A `course` consists of a `cno` (course code), a `title`, a prerequisite hierarchy (via `prereq`), and all the students who have registered for the course (via `takenBy`). A course may have several `projects`. A `student` has a `sno` (student number), a `name` and a list of `qualified` courses. Each `project` has a `pno` (project number), a `ptitle` (title) and required courses (`required`). Its DTD graph, a 3-cycle graph, is shown in Fig. 1 (a). □

2.2 XPath and Extended XPath

XPath. We consider a class of XPath queries [10] that supports recursion (descendant-or-self), union and rich qualifiers, given as follows.

$$p \quad ::= \quad \epsilon \quad | \quad A \quad | \quad * \quad | \quad p/p \quad | \quad //p \quad | \quad p \cup p \quad | \quad p[q]$$

where ϵ, A and $*$ denote the empty path, a label and a wildcard, respectively; '$\cup$', '/' and '//' are *union*, *child-axis* and *descendant-or-self-axis*, respectively; and q is a *qualifier*, defined as

$$q \quad ::= \quad p \quad | \quad text() = c \quad | \quad \neg q \quad | \quad q \wedge q \quad | \quad q \vee q$$

where c is a constant, and p is defined above.

An XPath query p, when evaluated at a *context node* v in an XML tree T, returns the set of nodes of T reachable via p from v, denoted by $v[\![p]\!]$. In particular, $v[\![p_1[q]]\!]$ consists of nodes reachable via p_1 from v that satisfy the qualifier $[q]$. More specifically, a node v' satisfies the qualifier $[q]$ as follows: the atomic predicate $[p]$ holds at v' iff $v'[\![p]\!]$ is nonempty, i.e., there exists a node reachable via p from v'; and $[\mathit{text}() = c]$ is true iff $v.\mathit{val}$ equals the constant c. The boolean operations are self-explanatory. We also use $\emptyset$ to denote a special query, which returns the empty set over all XML trees, with $\emptyset \cup p$ equivalent to p and $p/\emptyset/p'$ equivalent to $\emptyset$.

This class of XPath queries properly contains branching path queries studied in [27] and tree pattern queries (see, e.g., [3]). In the sequel, we refer to this class of queries simply as XPath.

Example 2.2: Consider two XPath queries.

Q_1 = `dept//project`
Q_2 = `dept/course[//prereq/course[cno="cs66"]` $\wedge$ $\neg$`//project`
        $\wedge$ $\neg$ `takenBy/student/qualified//course[cno = "cs66"]]`

Over an XML tree of the `dept` DTD of Fig.1, query Q_1 is to find all course-related projects, and Q_2 is to find courses that (1) have a prerequisite cs66, (2) have no project related to them or to their prerequisites, but (3) have no student who registered for the course and took cs66. □

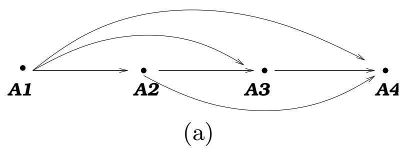
Extended XPath. As will be seen in the next section, to translate XPath queries over a (possibly recursive) DTD to SQL, we first rewrite XPath queries to expressions in an extension of XPath, referred to as *extended* XPath *expressions* and syntactically defined as follows:

$$\begin{array}{lcl} E & ::= & \epsilon \;\mid\; A \;\mid\; X \;\mid\; E/E \;\mid\; E \cup E \;\mid\; E^* \;\mid\; E[q], \\ q & ::= & E \;\mid\; \mathit{text}() = c \;\mid\; \neg q \;\mid\; q \wedge q \;\mid\; q \vee q. \end{array}$$

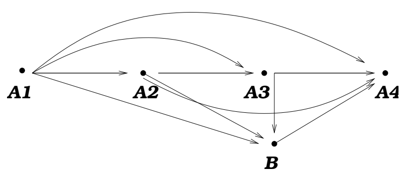
where X is a variable, and E^* denotes the Kleene closure of E.

Observe that an expression E without any variable is a query in *regular* XPath proposed in [34]. Regular XPath expressions are the "least upper bound" of XPath and regular expressions. They differ from XPath queries in that, first, they support general Kleene closure E^* as opposed to restricted recursion '//', and second, they do not allow wildcard $*$ and descendant '//'. They extend regular expressions by supporting qualifiers. The motivation for using E^* instead of '//' is twofold. First, the expressive power of E^* is required for encoding both DTD recursion and XPath recursion. As will be seen shortly, with E^* one can define a finite representation of (possibly infinite) matching paths of an XPath query over a recursive DTD. Second, E^* "instantiates" // with paths in the DTD. In a nutshell, E takes a union of all matching simple cycles of // and E^* then applies the Kleene closure to the union; each of these paths can then be mapped to a sequence of relations connected with joins. The semantics of evaluating E over an XML tree is similar to its XPath counterpart.

An *extended* XPath *query* Q is a sequence of equations of the form $X_i = E_i$, where for $i \in [1, k]$, X_i is a variable, E_i is an extended XPath expression, and X_i does not appear in E_j if $i < j$. Intuitively, the equations specify bindings of variables and sub-queries. It can be easily verified that Q is equivalent to a sequence of equations of the form $X_i = E'_i$, where E'_i is a regular XPath query, i.e., an extended XPath expression without variables. The semantics of evaluating Q over



(a)



(b)

Figure 2: Example DTD graphs.

an XML tree is therefore straightforward: for i from k downward to 1, evaluate E_i and substitute the result of E_i for X_{i-1} in E_{i-1}.

The use of variables in extended XPath is to represent a number of matching paths in polynomial that would otherwise take exponential time in regular XPath and would not be expressible in XPath. To illustrate this, let us consider the following example, taken from [13], which shows that representing a nondeterministic finite state automaton with a regular expression takes at least exponential time, even if the automaton is non-recursive.

Example 2.3: Consider a DTD D_1 for which the DTD graph consists of (a) nodes A_i for $i \in [1, n]$, where the root is A_1, (b) edges (A_i, A_j) for all $i, j \in [1, n]$ and $i < j$. Figure 2(a) shows such a DTD graph for $n = 4$. This DTD is contained in another DTD D_2, shown in Fig. 2(b), which, in addition, has a node B and moreover, edges (B, A_n) and (A_i, B) for $i < n$. Note that these DTD graphs are acyclic, i.e., they are non-recursive. There is a natural mapping σ_0 from instances of D_1 to instances of D_2 such that for any document T that conforms to D_2, from σ_0 an XML document V can be derived such that V conforms to D_1 and moreover, (a) the root r_v of V maps to the root r_t of T, and (b) for any element u in V that is reached from r_v via a path ρ, it is mapped to an element $\sigma_0(u)$ that is reachable from r_t via the same path ρ.

Now consider a query $Q = //A_n$ posed on V that is to find all A_n nodes in V. Suppose that we want to find an equivalent query Q' that, when posed on T, returns the same result as Q on V. The query Q' is then to find all A_n nodes in T that are reachable from the root without going through any B node in T. One can verify the following. (1) The query Q' is not expressible in the XPath fragment given above. (2) Although query Q' is expressible in regular XPath, it takes necessarily $O(2^n)$ space (the proof for this follows from the argument of [13]). (3) The query Q' can be expressed in extended XPath in polynomial space, as follows.

$$X_{(1,4)} = A_4 \cup A_2/X_{(2,4)} \qquad X_{(2,4)} = A_4 \cup A_3/X_{(3,4)} \qquad X_{(3,4)} = A_4$$

As we will seen shortly, we may encounter such queries in XPath query translation. This motivates us to use extended XPath instead of regular XPath and XPath. □

2.3 Mapping DTDs to a Database Schema

We next review shredding of XML data into relations. We focus on a DTD-based approach since it is supported by most RDBMS [23, 35, 38].

We adopt the shared-inlining technique of [43]. In a nutshell, the inlining algorithm partitions a DTD graph G_D into subgraphs, $G_1, G_2, \cdots$ such that any A-node is represented in exactly one subgraph and there is no edge labeled '$*$' in any subgraph. Each subgraph G_i is mapped to a relation schema R_i. Each relation schema has a key attribute ID. The edges from a subgraph G_i to a subgraph G_j are specified using *parentId* in the corresponding relation schema R_j. If a

F	T
-	d_1

(a) R_d

F	T
d_1	c_1
c_1	c_2
c_2	c_3
p_1	c_4
s_2	c_5

(b) R_c

F	T
c_1	s_1
c_1	s_2

(c) R_s

F	T
c_2	p_1
c_4	p_2

(d) R_p

Table 1: A database encoding an XML tree of the `dept` DTD

subgraph G_j has more than one incoming edge, say from G_i and G_k, a *parentCode* attribute is introduced into the relation schema R_j indicating the parent code of the R_j tuples.

We use $\tau : D \rightarrow \mathcal{R}$ to denote a mapping from DTD D to a relational database schema $\mathcal{R}$, which consists of a set of relation schemas. Observe that from τ one can easily derive a *data mapping*, denoted by τ_d, from XML trees of D to instances of $\mathcal{R}$.

To simplify the discussion we assume that the mapping τ maps each element type A to a relation R_A in $\mathcal{R}$, which has three columns F (from, i.e., *parentId*), T (to, i.e., ID) and V (the value of all other attributes). Intuitively, in a database $\tau_d(T)$ representing an XML tree T, each R_A tuple (f, t, v) represents an edge in T from a node f to an A-element t which may have a text value v, where t and f are denoted by the node IDs in T and are thus *unique* in the database, and v is '_' in the absence of text value at t. In particular, $f =$ '_' if and only if f is the root of T. This assumption does not lose generality: our query translation techniques can be easily extended to cope with mappings without this restriction.

Example 2.4: With the shared-inlining technique, the DTD graph G_D of Fig. 1 (a) is partitioned into four subgraphs rooted at *dept*, *course*, *project*, and *student*, respectively (see Fig. 1 (b)). It is mapped to a relational database schema $\tau(D)$ consisting of four corresponding relation schemas, R_d, R_c, R_p and R_s:

R_d(F, T)
R_c(F, T, cno, title, prereq, takenBy, parentCode)
R_s(F, T, sno, name, qualified)
R_p(F, T, pno, ptitle, required)

A sample database is shown in Table 1, which only shows F and T attributes. From Table 1 one can find paths in the XML tree of the `dept` DTD, e.g., $d_1.c_1.c_2.c_3$ and $d_1.c_1.c_2.p_1.c_4.p_2$. □

3 Overview: From XPath to SQL

The query translation problem from XPath to SQL is stated as follows. Let $\tau : D \rightarrow \mathcal{R}$ be a mapping from a DTD D to a relational schema $\mathcal{R}$, and τ_d be the corresponding data mapping from XML trees of D to the relational instance of $\mathcal{R}$. The problem is to find an algorithm that, given an XPath query Q, effectively computes an equivalent sequence of relational queries Q' such that for any XML tree T of the DTD D, $Q(T) = Q'(\tau_d(T))$.

In this section we first review the approach proposed by [27], the only solution published so far for the query translation problem in the presence of recursive DTDs. To overcome its limitations, we then propose a new approach and outline it in this section; detailed algorithms are provided in the next two sections. Finally, we show that our algorithms also provides a solution to query answering for certain XML views of XML data.

```
1.  with
2.  $R$ ($F, T, Rid$) as (
3.    (select $R_c.F$, $R_c.T$, $Rid$('c') from $R_d$, $R_c$)
4.    where $R_c.T = R_d.F$
5.    union all
6.    (select $R.F$, $R_c.T$, $Rid$('c')
7.            from $R, R_c$ where $R.T = R_c.F$ and $Rid$ = 'c')
8.    union all
9.    (select $R.F$, $R_s.T$, $Rid$('s')
10.           from $R, R_s$ where $R.T = R_s.F$ and $Rid$ = 'c')
11.   union all
12.   (select $R.F$, $R_c.T$, $Rid$('c')
13.           from $R, R_c$ where $R.T = R_c.F$ and $Rid$ = 's')
14.   union all
15.   (select $R.F$, $R_p.T$, $Rid$('p')
16.           from $R, R_p$ where $R.T = R_p.F$ and $Rid$ = 'c')
17.   union all
18.   (select $R.F$, $R_c.T$, $Rid$('c')
19.           from $R, R_c$ where $R.T = R_c.F$ and $Rid$ = 'p'))
```

Figure 3: The SQL statement generated by SQLGen-R

3.1 Linear Recursion of SQL'99

The algorithm of [27], referred to as SQLGen-R, handles recursive path queries over recursive DTDs based on the SQL'99 recursion operator. In a nutshell, given an input path query, SQLGen-R first derives a *query graph*, G_Q, from the DTD graph to represent all matching paths of the query in the DTD graph. It then partitions G_Q into strongly-connected components $c_1, \ldots, c_n$, sorted in the top-down topological order. It generates an SQL query Q_i for each c_i in the topological order, and associates Q_i with a temporary relation TR_i such that TR_i can be directly used in later queries Q_j for $j > i$. The sequence $TR_1 \leftarrow Q_1; \ldots; TR_n \leftarrow Q_n$ is the output of the algorithm. If a component c_i is cyclic, the SQL query Q_i is defined in terms of the *with...recursive* operator. More specifically, it generates two parts from c_i: an *initialization* part and a *recursive* part. The initialization part captures all "incoming edges" into c_i. The recursion part first creates an SQL query for each edge in c_i, and then encloses the union of all these (edge) queries in a *with...recursive* expression. It should be noted that if c_i has k edges, the query Q_i actually calls for a fixpoint operator $\phi(R, R_1, R_2, \cdots R_k)$ with $k + 1$ input relations, defined as follows:

$$\begin{aligned} R^0 &\leftarrow R \\ R^i &\leftarrow R^{i-1} \cup (R^{i-1} \bowtie_{C_1} R_1) \cup \cdots \cup (R^{i-1} \bowtie_{C_k} R_k) \end{aligned} \tag{1}$$

where R^0 corresponds to the initialization part, R_j corresponds to an SQL query coding an edge in c_i, and C_j indicates additional conditions associated with the join, for each $j \in [1, k]$.

Example 3.1: Recall the mapping given in Example 2.4 from the `dept` DTD to the relational schema $\mathcal{R}$ consisting of R_s, R_c, R_p, R_d, and the XPath query Q_1 = `dept//project` given in Example 2.2, which, over the DTD graph of Fig. 1 (b), indicates $R_d//R_p$. Given Q_1 and the DTD graph of Fig. 1 (b), the algorithm SQLGen-R finds a strongly-connected component $(R_c//R_p)$ having 3 nodes and 5 edges, and produces a single SQL query using a *with...recursive* expression, as shown in Fig. 3. More specifically, the initial part of the recursion is given in lines 3-4, while

iteration	F	T	*Rid*
0	d_1	c_1	‘c’
1	c_1	c_2	‘c’
	c_1	s_1	‘s’
	c_1	s_2	‘s’
2	c_2	c_3	‘c’
	c_2	p_1	‘p’
	s_2	c_5	‘c’
3	p_1	c_4	‘c’
4	c_4	p_2	‘p’

Table 2: An output of SQLGen-R at each iteration.

the recursion part is lines 6-19. Each edge in the graph Fig.1 (b) is translated into a select statement. Observe that in the select statement, it uses *Rid* to keep track of where the tuples in the result relation R come from. For example, the select statement for the edge $R_c \rightarrow R_c$ (lines 6-7) inserts a tuple into the result relation R with its F and T values in addition to a *Rid* value ‘c’ indicating that it is from relation R_c. The usage of *Rid* is to join right parent/child tuples. As line 10 shows, in the select statement for the edge $R_c \rightarrow R_s$, it needs to join with tuples in R that is originally from R_c (*Rid* = ‘c’). Similarly for $R_s \rightarrow R_c$, $R_c \rightarrow R_p$, and $R_p \rightarrow R_c$ (lines 12-13, 15-16 and 18-19, respectively). When evaluated over the relational database of Table 1, the query of Fig. 3 returns the result shown in Table 2. Using a selection on *Rid* = ‘p’ on Table 2, one can find that p_1 and p_2 are the descendants of p. □

Observe the following about the query of Fig. 3. First, it actually requires a fixpoint operator that takes 4 relations as input. As we have remarked in Section 1, $\phi(R, R_1, R_2, \cdots R_k)$ is a high-end feature that few RDBMS support. Although theoretically one can encode this in terms of a LFP $\Phi(R)$ that takes a single input relation and is supported by most commercial RDBMS, the coding introduces space overhead. Second, it is a complex query consisting of 5 joins and 5 unions. That is, each iteration of the fixpoint computation needs to compute 5 joins and 5 unions. Third, *with...recursive* is treated as a black box. In this example, all 5 relations join the result relation R in the center, which forms a *star* shape. The relation in the center keeps growing, but one can do little to optimize the operations inside the *with...recursion* expression.

3.2 A New Approach

To overcome the limitations of the previous approach, we propose a new approach to translating XPath queries to SQL, based on a notion of extended XPath expressions and the simple LFP operator $\Phi(R)$. Below we first review the simple LFP operator. We then outline our approach.

The LFP operator. The LFP operator $\Phi(R)$ takes a single input relation R, as shown below:

$$\begin{aligned} R^0 &\leftarrow R \\ R^i &\leftarrow R^{i-1} \cup (R^{i-1} \bowtie_C R^0) \end{aligned} \tag{2}$$

where C is a condition associated with the join. This LFP operator is already supported by most commercial products, e.g., by Oracle and IBM DB2 are shown in Fig. 4.

To illustrate how the LFP operator handles Kleene closure, consider an extended XPath expression $(A_1/\cdots/A_n)^*$ representing a simple cycle $A_1 \rightarrow \cdots \rightarrow A_n \rightarrow A_1$. This simple

```
LFP $\Phi(R)$ in Oracle
    select $F$, $T$ from $R$ connect by $F$ = prior $T$

LFP $\Phi(R)$ in DB2
    1.  with
    2.  $R_\Phi(F,T)$ as (
    3.  (select $F$, $T$ from $R$)
    4.  union all
    5.  (select $R_\Phi.F$, $R.T$ from $R_\Phi, R$ where $R_\Phi.T = R.F$))
```

Figure 4: Implementation of LFP in Oracle and DB2

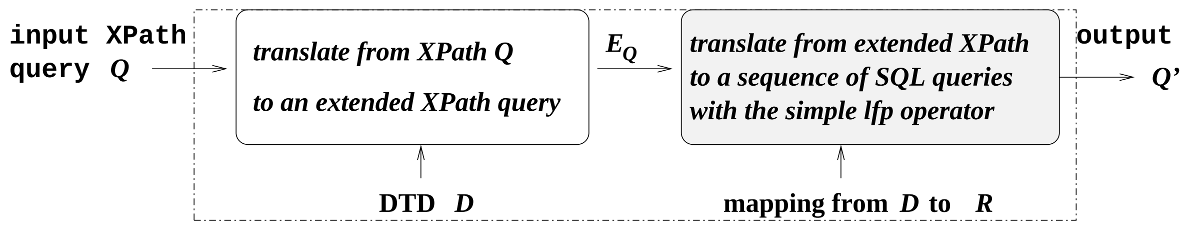
extended XPath expression can be rewritten into $\Phi(R)$ (Eq. (2)) by letting $R \leftarrow \Pi_{R_1.F, R_n.T}(R_1 \bowtie R_2 \bowtie \cdots \bowtie R_n)$, where the projected attributes are taken from the attributes F (from) and T (to) in relations R_1 and R_n, respectively. The join between R_i/R_j is expressed as $R_i \bowtie_{R_i.T=R_j.F} R_j$, i.e., it returns R_i tuples that *connect* to R_j tuples. In general, we rewrite E^* to $\Phi(R)$, where R is a temporary relation associated with a query that encodes E.

A new approach for query translation. Based on the LFP operator $\Phi(R)$ and extended XPath queries, we propose a new framework for translating XPath to SQL. As depicted in Fig. 5, the framework translates an input XPath query Q to SQL in two steps. First, it rewrites Q over a (possibly recursive) DTD D to an equivalent extended XPath query E_Q over any DTD D' that contains D, i.e., the DTD graph of D is a subgraph of the DTD graph of D'. The query E_Q has the form $(X_1 = E_1, \ldots, X_k = E_k)$ as mentioned above. Second, it rewrites E_Q into an equivalent sequence Q' of SQL queries based on a mapping $\tau : D \to \mathcal{R}$, and using the LFP operator to handle Kleene closure. The choice of extended XPath in the first step is motivated by the following reasons. As remarked earlier, the Kleene closure of extended XPath allows us to instantiate '//' of XPath, and capture recursion in XPath and DTD recursion in a uniform framework. Furthermore, as illustrated in Example 2.3, the use of variables allows us to extract common sub-queries and thus avoid the exponential lower bound of translation to regular XPath.

In contrast to the approach of [27], this framework introduces more opportunities for optimization, as illustrated by the example below.

Example 3.2: Let us consider again evaluating the XPath query $Q_1 =$ `dept//project` over the `dept` DTD of Fig. 1, in the same setting as in Example 3.1. Our translation algorithms first translate Q_1 to an extended XPath query $E_{Q_1} = (X_{Q_1} = R_d/R_c/X^*/R_p,\ X = R_c \cup R_s/R_c \cup R_p/R_c)$. It then rewrites E_{Q_1} to a sequence of SQL queries (written in relational algebra):

$$\begin{array}{rcl}
R_{cc} & \leftarrow & R_c \\
R_{csc} & \leftarrow & \Pi_{R_s.F, R_c.T}(R_s \bowtie_{R_s.T=R_c.F} R_c) \\
R_{cpc} & \leftarrow & \Pi_{R_p.F, R_c.T}(R_p \bowtie_{R_p.T=R_c.F} R_c) \\
R & \leftarrow & R_{cc} \cup R_{csc} \cup R_{cpc} \\
R_\gamma & \leftarrow & \Phi(R) \cup \Pi_{T,T}(R_c) \\
R_f & \leftarrow & \Pi_{R_d.T, R_p.T}(R_d \bowtie_{R_d.T=R_c.F} R_c \bowtie_{R_c.T=R_\gamma.F} R_\gamma \bowtie_{R_\gamma.T=R_p.F} R_p)
\end{array}$$


input XPath query Q
translate from XPath Q to an extended XPath query
E_Q
translate from extended XPath to a sequence of SQL queries with the simple lfp operator
output Q'
DTD D
mapping from D to R
Figure 5: Translation from XPath to SQL

The above SQL sequence is the output of our algorithms. Contrast this with Example 3.2. While our SQL queries use 3 unions and 5 joins in total, they are evaluated once only, instead of once in each iteration of the LFP computation. In other words, we pull join/union out from the black box of *with...recursive*. This not only gives us more opportunities to optimize join/union, but also allows us to push selection conditions into the LFP operator, along the same lines as the LFP optimization by distribution of selections suggested by [2], as will be illustrated in Section 5. □

In addition to optimization opportunities, the first step of the framework in fact provides a solution to query answering for certain XML views of XML data.

Query answering for XML views. Consider a class of GAV mappings $\sigma : D_1 \to D_2$, where D_1, D_2 are target and source DTDs, respectively (see, e.g., [22, 31] for GAV mappings), such that for any instance T of D_2, σ defines a view V such that for any XML element $v \in V$, $\sigma(v)$ is contained in an XML element u in T, and V is a sub-structure of T. For instance, the mapping σ_0 given in Example 2.3 is such a mapping from the DTD of Fig. 2(a) to the DTD of Fig. 2(b). Such views are found in many applications, e.g., access control for XML [14] where one only wants to reveal parts of T to authorized users, or data integration [31] where part of the source is migrated to the target. In these applications V is often virtual.

Now consider an XPath query posed on V. We want to answer the query without materializing V. This highlights the need for a *query answering* algorithm that, given an XPath query Q posed on D_1, effectively computes a query Q' on D_2 such that $Q(V) = Q'(T)$.

This query answering problem is, however, nontrivial. Indeed, consider query Q given in Example 2.3, which is posed on the view V. The equivalent query Q' on the source T is to find all A_n nodes reachable from A_1 without going through any B nodes. As shown in Example 2.3, this query is not expressible in XPath, and although it is definable in regular XPath, it is necessarily of exponential size. As recently observed in [15], XPath is not closed under query rewriting and although regular XPath is closed, it incurs an exponential lower bound for rewritten queries.

In contrast, we show that the translation algorithm of the step 1 of our framework provides a solution to the query answering problem for the class of GAV XML views described above. Indeed, given any XPath query Q posed on D_1, the algorithm effectively computes Q' in extended XPath in *polynomial time*. Furthermore, the query Q' has the following property. For any D_2 that contains D_1, define a mapping σ from instances of D_1 to instances of D_2 that, given any instance T of D_2, derives an instance V of D_1 such that V is a subtree of T: σ maps the root r_v of V to the root r_t of T, each A element v of V to an A element $\sigma(v)$ of T such that if v is reached from r_v via a path ρ, then $\sigma(v)$ can reached from r_t via the same path ρ. Then $Q(V) = Q'(T)$. We say that Q' is *equivalent to* Q over all DTDs D_2 that contain D_1. Furthermore, we say that Q' is *equivalent to* Q *w.r.t. two element types* A, B over all DTDs D_2 that contains D_1 if when evaluated at any A element v in V, the set of B elements returned by

Q is the same as the set of B elements returned by Q' when evaluated at $\sigma(v)$ in T.

In Section 4 we present a translation algorithm to show that every XPath query Q over a (recursive) DTD D can be rewritten to an extended XPath query E_Q that is equivalent to Q over all DTDs containing D. Then, we provide another algorithm in Section 5 to show that the simple LFP operator $\Phi(R)$ suffices to handle general Kleene closure in an extended XPath query E_Q.

4 From XPath to Extended XPath

In this section, we present an algorithm for rewriting an XPath query Q over a (recursive) DTD D to an extended XPath query E_Q that is equivalent to Q' over all DTDs containing D. We first develop an algorithm for handling the descendant-axis ('//') of XPath, and then give the translation algorithm for the XPath fragment defined in Section 2.

4.1 Translation of the Descendant Axis

Consider an XPath query $Q = A//B$ over a DTD D. The query, when evaluated at an A-element v in an instance V of D, is to find all B descendants of v. We want to find an extended XPath query Q', denoted by $\mathsf{rec}(A, B)$, that is equivalent to Q over all DTDs that contain D.

An algorithm is given by Tarjan in [45] that, given a graph G_D and two nodes $A, B \in G_D$, finds a regular expression which represents the set of all paths in G_D from A to B. We sketch Tarjan's algorithm [45] in Fig. 6, and denote it by CycleE as it is based on cycle expansion. Let $G_D = (N, E)$, where N is the set of nodes and E is the set of edges of G_D. Following the notations of [45], we associate the nodes in G_D with numbers from 1 to $n = |N|$, and use a regular expression $M[i, j, k]$ to maintain all possible paths from node i to node j via nodes whose numbers are less than or equal to k, where k can be zero indicating a "path" via no nodes in G_D. Algorithm CycleE first initializes all $M[i, j, 0]$ (line 1-7). It then expands $M[i, j, k]$ for all i, j by incrementing k, i.e., by inspecting $M[i, k-1, k]$ and $M[k, j, k]$ while including all possible cycles, i.e., $M[k, k, k-1]^*$, at node k (lines 8-13).

The regular expression $M[1, n, n]$ returned by Algorithm CycleE precisely represents all paths from node A to node B, where A and B are numbered 1 and n, respectively. Unfortunately, the algorithm takes exponential time and exponential space.

Lemma 4.1: *Given a graph G_D with n nodes and nodes A, B in G_D,* CycleE *finds a regular expression capturing all paths in G_D from node A to B in $\Theta(n^3 2^n)$-time and $\Theta(n^2 2^n)$-space.* □

Proof: The correctness of algorithm was verified in [45]. The upper bound is due to line 12 in CycleE (Fig. 6), which copies sub-expressions and concatenates them into one. In fact, it is already shown in [13] that the bounds given above are also the lower bounds for converting nondeterministic finite state (NFA) to regular expressions; as a result, when the graph D_G is treated as a NFA with A as the start state and B the final state, any regular expressions characterizing the NFA have the lower bounds given above. □

Recall the definition of extended XPath expression and extended XPath query. We show that one can find a query $\mathsf{rec}(A, B)$ representing all paths from A to B in *low polynomial time.* Indeed, we present a mildly modified CycleE, denoted by $\mathsf{CycleE_X}$, that computes $\mathsf{rec}(A, B)$. The algorithm uses the following variables. (a) $M[i, j, k]$ is an extended XPath expression representing all possible paths from node i to node j via nodes no larger than k. (b) Variable $X[i, j, k]$

```
Algorithm CycleE($G_D, A, B$)
Input: a graph $G_D$ and two nodes $A$ and $B$ in $G_D$.
output: a regular expression representing all paths from $A$ to $B$ in $G_D$.
1.  for $i = 1$ to $n$ do
2.      for $j = 1$ to $n$ do
3.          if $i = j$
4.          then $M[i, j, 0] := \emptyset$;
5.          else if $i \neq j$ and $(i, j) \in E(G_D)$
6.               then $M[i, j, 0] := i/j$;
7.               else $M[i, j, 0] := \emptyset$;
8.  for $k = 1$ to $n$ do
9.      for $i = 1$ to $n$ do
10.         for $j = 1$ to $n$ do
11.             if $M[i, k, k-1] \neq \emptyset$ and $M[k, j, k-1] \neq \emptyset$
12.             then $M[i, j, k] := (M[i, j, k-1]) \cup (M[i, k, k-1]/M[k, k, k-1]^*/M[k, j, k-1])$;
13.             else $M[i, j, k] := M[i, j, k-1]$;
14. return $M[A, B, n]$;
```

Figure 6: CycleE (Tarjan's Algorithm for finding regular expressions)

```
Algorithm CycleE$_X$($G_D, A, B$)
Input: a DTD graph $G_D$ and two nodes $A$ and $B$ in $G_D$.
output: an extended XPath query rec($A, B$) representing all paths from $A$ to $B$ in $G_D$.
1.  for $i = 1$ to $n$ do
2.      for $j = 1$ to $n$ do
3.          if $i = j$
4.          then $M[i, j, 0] := \emptyset$;
5.          else if $i \neq j$ and $(i, j) \in E(G_D)$
6.               then $M[i, j, 0] := i/j$;
7.               else $M[i, j, 0] := \emptyset$;
8.  for $k = 1$ to $n$ do
9.      for $i = 1$ to $n$ do
10.         for $j = 1$ to $n$ do
11.             if $M[i, k, k-1] \neq \emptyset$ and $M[k, j, k-1] \neq \emptyset$
12.             then $M[i, j, k] :=$ '$X[i, j, k-1] \cup X[i, k, k-1]/X[k, k, k-1]^*/X[k, j, k-1]$';
13.             else $M[i, j, k] :=$ '$X[i, j, k-1]$';
14. rec($A, B$) $:= \{(X[i, j, k] = M[i, j, k]$, where $M[i, j, k] \neq \emptyset \mid i, j, k \in [0, n]\}$;
15. optimize rec($A, B$) by removing redundant equations and return rec($A, B$).
```

Figure 7: CycleE$_X$ for extended XPath expressions

indicates equation $X[i, j, k] = M[i, j, k]$ in the output. (c) $X[k, k, k-1]^*$ indicates equation $X[k, k, k-1] = cycle(M[k, k, k-1])$, which represents cycles at node k. The algorithm is shown in Fig. 7. The initialization part (lines 1-7) is the same as its counterpart in CycleE. In contrast to CycleE, $M[i, j, k]$ is represented as an expression (string) with only at most four operators and four variables rather than concatenating four expressions, by capitalizing on variables (lines 8-13). The length of each $M[i, j, k]$ is thus constant. Finally, we construct extended XPath query rec(A, B) by listing equations (i.e., $X_i = E_i$, where X_i is a variable and E is an extended XPath expression) $X[i, j, k] = M[i, j, k]$ in the order of k, and return the whole ordered set rec(A, B) as the output, where variable $X[A, B, n]$ represents the final result (lines 14-15). In line 15, the following redundant equations are pruned from rec(A, B): 1) $X[i, j, k] = \emptyset$; 2)

$X[i, j, k]$ = '$X[i', j', k']$'; and 3) the variables that do not contribute to processing the variable $X[A, B, n]$.

Example 4.1: Consider again the query $//A_n$ on the DTD graph D_1 of Fig. 2(a). Starting from A_1 and ending with A_4 ($A_1//A_n$), for $n = 4$, CycleE$_X$ returns an extended XPath query as follows (suppose nodes A_1, A_2, A_3, and A_4 in D_1 are associated with numbers 1, 2, 3, and 4, respectively):

$$\begin{aligned} X[1,3,2] &= \text{`}X[1,3,1] \cup X[1,2,1]/X[2,3,1]\text{'}, \quad (3)\\ X[1,4,2] &= \text{`}X[1,4,1] \cup X[1,2,1]/X[2,4,1]\text{'}, \quad (4)\\ X[1,4,4] &= \text{`}X[1,4,2] \cup X[1,3,2]/X[3,4,2]\text{'}, \quad (5)\end{aligned}$$

where $X[1, 2, 1] = 1/2$, $X[1, 3, 1] = 1/3$, $X[1, 4, 1] = 1/4$, $X[2, 3, 1] = 2/3$, $X[2, 4, 1] = 2/4$, and $X[3, 4, 2] = 3/4$.[1] The output of CycleE$_X$ produces an extended XPath query that contains 3 "$\cup$"-operators and 6 "/"-operators. Note: the "/" appearing in Eq. (3), Eq. (4), and Eq. (5) is used to concatenate two variables, and is not a "/"-operator in the extended XPath expression. For example, consider '$X[1, 2, 1]/X[2, 3, 1]$' (Eq. (3)). Here, $X[1, 2, 1] = 1/2$ indicates a path from 1 to 2, and $X[2, 3, 1] = 2/3$ indicates a path from 2 to 3. Node 2 appears in both $X[1, 2, 1]$ and $X[2, 3, 1]$, and concatenates the two variables. Similarly, $X[1, 2, 1]/X[2, 3, 1]$ indicates an extended XPath expression 1/2/3.

In contrast, CycleE gives an extended XPath expression of $X[1, 4, 4] = 1/4 \cup 1/2/4 \cup (1/3 \cup 1/2/3)/4$ with 3 "$\cup$"-operators and 7 "/"-operators. □

Example 4.1 illustrates how CycleE$_X$ works. Next, we also show CycleE (polynomial) outperforms CycleE$_X$ (exponential), in terms of the number the "/"-operators.

Example 4.2: Consider the query $//A_n$ on the DTD graph D_n, which consists of: (a) nodes A_i for $i \in [1, n]$, where the root is A_1; (b) edges (A_i, A_j) for all $i, j \in [1, n]$ and $i < j$. Starting from A_1 and ending with A_n ($A_1//A_n$), CycleE$_X$ returns an extended XPath query (a list of equations) with $\Theta(n^2)$ "/"-operators, while CycleE gives an extended XPath expression with $\Omega(2^n)$ "/"-operators. Indeed, when CycleE$_X$ is used, only one "/"-operator, appearing on the right side of the equation for $X[1, i, j]$, where $1 \le j < i \le n$, will be executed. In total, there are $n \cdot (n - 1)/2 \in \Theta(n^2)$ "/"-operators. Consider CycleE. Let $f(n)$ be the number of "/"-operators in the output of CycleE on the input DTD graph D_n. We can establish the following recursive relationship from line 12 in Algorithm CycleE:

$$\begin{aligned} f(2) &= 1;\\ f(3) &= f(2) + 2;\\ f(4) &= f(2) + f(3) + 3;\\ \dots &\;\dots\; \dots\\ f(n) &= f(2) + f(3) + \dots + f(n-1) + (n-1);\end{aligned}$$

[1]Based on the pruning rule of 2), the $X[1, 2, 1]$ = '$X[1, 2, 0]$' is pruned and $X[1, 2, 1] = 1/2$ produced by Algorithm CycleE$_X$ is used. The equation of $X[2, 4, 3]$ = '$X[2, 4, 2] \cup X[2, 3, 2]/X[3, 4, 2]$' will be pruned following the pruning rule of 3), because it does not contribute to the processing of $X[1, 4, 4]$.

Thus $f(n) = n + (n-1) + 2 \cdot (n-2) + 2^2 \cdot (n-3) + ... + 2^{i-1} \cdot (n-i) + ... + 2^{n-4} \cdot 3 \in \Omega(2^n)$. □

Moreover, in contrast to CycleE, Algorithm $\mathsf{CycleE_X}$ has the following nice properties.

Theorem 4.1: *Given a* DTD *D with n element types and element types A, B in D,* $\mathsf{CycleE_X}$ *finds a query* $\mathsf{rec}(A, B)$ *in extended* XP*ath in $O(n^3 log\ n)$ time, and moreover, when evaluated at any A-element,* $\mathsf{rec}(A, B)$ *is equivalent to $//B$ for all* DTD*s that contain D.* □

Proof: For the complexity, $\mathsf{CycleE_X}$ computes at most $n^3 + n$ equations, and each equation is of $O(log\ n)$ size (for encoding the four variables, and there are $n^3 + n$ variables in total). Each step of the inner-most loop takes at most $O(log\ n)$ time. From this the complexity bound follows.

We next show that $\mathsf{rec}(A, B)$ is equivalent to $//B$ for all DTDs that contain D. More specifically, let D' be an arbitrary DTD that contains D, and σ be the mapping that, given any instance T of D', extracts a subtree V of T that is an instance of D, as specified in Section 3. Let v be an A element in V, where σ maps v to an A element $\sigma(v)$ in T. We need to show that a node u' can be reached from $\sigma(v)$ via $\mathsf{rec}(A, B)$ in T iff there exists a node u in V such that $u' = \sigma(u)$ and u can be reached from v via $\mathsf{rec}(A, B)$ in V.

To show this, first observe that a regular expression $\mathcal{X}(A, B)$ can be derived from $\mathsf{rec}(A, B)$ by removing variables as described in Section 2. We claim the following: ρ is a path from A to B in D iff ρ is a word in $\mathcal{X}(A, B)$. This can be easily verified by showing ρ is a path from A to B without going through any node larger than k iff $\rho \in M[A, B, k]$, by induction on k.

This claim suffices. Indeed, for any XML tree T and any A element $\sigma(v)$ in T, a node u' can be reached from $\sigma(v)$ in T via $\mathsf{rec}(A, B)$ iff the path ρ from $\sigma(v)$ to u' in T is a word in $\mathcal{X}(A, B)$. By the claim and the definition of σ, this happens iff ρ is in D and there exists a node u in V such that $u' = \sigma(u)$ and u can be reached from v via the same path ρ in V. □

4.2 Translation Algorithm

We next present an algorithm for translating XPath queries of the fragment of Section 2 over a DTD D to extended XPath queries that are equivalent over all DTDs that contain D.

The algorithm, referred to as $\mathsf{XPathToEXp}$, is based on *dynamic programming*: for each sub-query p of the input query Q and each pair of element types A, B in D, it computes a local translation from XPath p to an equation $X_p(A, B) = \mathsf{x2e}(p, A, B)$, where $X_p(A, B)$ is a variable and $\mathsf{x2e}(p, A, B)$ is an extended XPath expression, such that $\mathsf{x2e}(p, A, B)$ is equivalent to p w.r.t. A and B over any DTD D' that contains D (recall the notion from Section 3). Composing the local translations one will get the rewriting $E_Q = \bigcup_{B \in D} X_Q(r, B)$ from Q, where r is the root type of D. For each local translation $\mathsf{x2e}(p, A, B)$ the algorithm "evaluates" p over the sub-graph of the DTD graph G_D rooted at A, substituting extended XPath expressions over element types for wildcard $*$ and descendants $//$, by incorporating the structure of the DTD into $\mathsf{x2e}(p, A, B)$. This also allows us to "optimize" the XPath query by capitalizing on the DTD structure: certain qualifiers in p can be evaluated to their truth values and thus be eliminated during the translation, just by checking the structure of D.

To conduct the dynamic-programming computation, Algorithm $\mathsf{XPathToEXp}$ uses the following variables. First, it works over a list L that is a postorder enumeration of the nodes in the parse tree of Q, such that all sub-queries of a sub-query p (i.e., its descendants of p in Q's parse tree) precede p in L. Second, all the element types of the DTD D are put in a list

```
Algorithm XPathToEXp
Input: an XPath query $Q$ over a DTD $D$.
Output: an extended XPath query $E_Q$ that is equivalent to $Q$ over all DTDs that contain $D$.
1.  compute the ascending list $L$ of sub-queries in $Q$;
2.  compute the list $N$ of all the types in $D$;
3.  for each $p$ in $L$ do
4.      for each $A, B$ in $N$ do
5.          $\mathsf{x2e}(p, A, B) := \emptyset$;   $\mathsf{reach}(p, A) := \emptyset$;
6.  for each $p$ in the order of $L$ do
7.    for each $A$ in $N$ do
8.      case $p$ of
9.      (1)  $\epsilon$:  $\mathsf{x2e}(p, A, B) := \epsilon$ for all $B \in N$;   $\mathsf{reach}(p, A) := \{A\}$;
10.     (2)  $B$:  if $B$ is a child type of $A$
11.                then $\mathsf{x2e}(p, A, B) := B$;  $\mathsf{reach}(p, A) := \{B\}$;
12.                else $\mathsf{x2e}(p, A, B) := \emptyset$;  $\mathsf{reach}(p, A) := \emptyset$;
13.     (3)  $*$:  for each child type $B$ of $A$ in $D$ do
14.                  $\mathsf{x2e}(p, A, B) := B$;   $\mathsf{reach}(p, A) := \mathsf{reach}(p, A) \cup \{B\}$;
15.     (4)  $p_1/p_2$:  for each $C$ in $\mathsf{reach}(p_1, A)$ and each $B$ in $\mathsf{reach}(p_2, C)$ do
16.                  $\mathsf{x2e}(p, A, B) := X_{p_1}(A, C)/X_{p_2}(C, B)$;   $\mathsf{reach}(p, A) := \mathsf{reach}(p, A) \cup \{B\}$;
17.     (5)  $\epsilon//p_1$:   /* $\mathsf{reach}(A, \epsilon//)$, $\mathsf{rec}(A, B)$ are precomputed, with $X_r(A, B) = \mathsf{rec}(A, B)$ */
18.          for each $C$ in $\mathsf{reach}(A, \epsilon//)$ and each $B$ in $\mathsf{reach}(C, p_1)$ do
19.            $\mathsf{x2e}(p, A, B) := X_r(A, C)/X_{p_1}(C, B)$;   $\mathsf{reach}(p, A) := \mathsf{reach}(p, A) \cup \{B\}$;
20.     (6)  $p_1 \cup p_2$:  $\mathsf{x2e}(p, A, B) := X_{p_1}(A, B) \cup X_{p_2}(A, B)$ for all $B \in D$;
21.                  $\mathsf{reach}(p, A) := \mathsf{reach}(p_1, A) \cup \mathsf{reach}(p_2, A)$;
22.     (7)  $p'[q]$:
23.          for each $B$ in $\mathsf{reach}(p', A)$ do
24.              RewQual $([q], B)$;     /* RewQual $([q], B)$ returns $\mathsf{x2e}([q], B, B)$ and $X_{[q]}(B, B)$ */
25.              if $\mathsf{x2e}([q], B, B) = [\epsilon]$     /* $[q]$ holds at $B$ */
26.              then $\mathsf{x2e}(p, A, B) := X_{p'}(A, B)$; $\mathsf{reach}(p, A) := \mathsf{reach}(p, A) \cup \{B\}$;
27.              else if $\mathsf{reach}([q], B) \neq [\emptyset]$     /* $[q]$ is not false at $B$ */
28.              then $\mathsf{x2e}(p, A, B) := X_{p'}(A, B)[X_{[q]}(B, B)]$; $\mathsf{reach}(p, A) := \mathsf{reach}(p, A) \cup \{B\}$;
29.     optimize $\mathsf{x2e}(p, A, B)$ by removing $\emptyset$ using $\emptyset \cup E = E$, $E_1/\emptyset/E_2 = \emptyset$;
30. $E_Q := \{X_Q = \bigcup_{B \in \mathsf{reach}(Q, r)} X_Q(A.B)\}$;     /* $r$ is the root of $D$ */
31. $E_Q := E_Q \cup \{X_p(A, B) = \mathsf{x2e}(p, A, B) \mid p \in L,\ A \in N, B \in N\} \cup \{\mathsf{rec}(A, B) \mid A \in N, B \in N\}$;
32. return $E_Q$;
```

Figure 8: Rewriting algorithm from XPath to extended XPath

N. Third, for each sub-query p in L and each pair of types A, B in N, we use $\mathsf{x2e}(p, A, B)$ to denote the *local translation* of p at A, which is an extended XPath expression. Furthermore, we use a variable $X_p(A, B)$ which will be used in the equation $X_p(A, B) = \mathsf{x2e}(p, A, B)$, such that $X_p(A, B)$ can be used instead of $\mathsf{x2e}(p, A, B)$ whenever the latter is needed. Finally, we also use $\mathsf{reach}(p, A)$ to denote the types in D that are *reachable* from A via p. Abusing this notation, we use $\mathsf{reach}([q], A)$ for a qualifier $[q]$ to denote whether or not $[q]$ can be evaluated to false at an A element, indicated by whether or not $\mathsf{x2e}([q], A, A)$ is $[\epsilon]$.

Algorithm XPathToEXp is given in Fig. 8. It computes E_Q as follows. It first enumerates the list L of sub-queries in Q and the list N of element types in D, as well as initializes $\mathsf{x2e}(p, A)$ to the special query $\emptyset$ and $\mathsf{reach}(p, A)$ to empty set for each $p \in Q$ and $A \in N$ (lines 1–5). Then, for each sub-query p in L in the topological order and each element type A in N, it computes the local translation $\mathsf{x2e}(p, A, B)$ (lines 6–28), bottom-up starting from the inner-most sub-query

```
Procedure RewQual ([q], A)
Input: an XPath qualifier $[q]$ and an element type $A$ in a DTD $D$.
Output: an extended XPath query $\mathsf{x2e}[q], A, A)$ equivalent to $[q]$ at $A$ elements over DTDs that contain $D$.
1.  case $[q]$ of
2.  (a) $[p_1]$:  XPathToEXp ($p_1$);    /* XPathToEXp computes $\mathsf{x2e}(p_1, A, B)$ and $\mathsf{reach}(p_1, A)$ */
3.            if $\mathsf{reach}(p_1, A) = \emptyset$
4.            then $\mathsf{x2e}([q], A, A) := \emptyset$;
5.            else if $\epsilon$ is in $\mathsf{x2e}(p_1, A, B)$
6.            then $\mathsf{x2e}([q], A, A) := \epsilon$;
7.            else for each $B \in \mathsf{reach}(p_1, A)$ do
8.                  $\mathsf{x2e}([q], A, A) := \mathsf{x2e}([q], A, A) \cup X_{p_1}(A, B)$;
9.  (b) $[text() = c]$:  $\mathsf{x2e}([q], A, A) := text() = c$;
10. (c) $[q_1 \wedge q_2]$:  RewQual ($q_1, A$);  RewQual ($q_2, A$);
11.             $\mathsf{x2e}([q], A, A) := \mathsf{optimize}(X_{[q_1]}(A, A) \wedge X_{[q_2]}(A, A))$;
12. (c)  $[q_1 \vee q_2]$:  RewQual ($q_1, A$);  RewQual ($q_2, A$);
13.             $\mathsf{x2e}([q], A, A) := \mathsf{optimize}(X_{[q_1]}(A, A) \vee X_{[q_2]}(A, A))$;
14. (e)  $[\neg q_1]$:  RewQual ($q_1, A$);
15.           $\mathsf{x2e}([q], A, A) := \mathsf{optimize}(\neg X_{q_1}(A, A))$;
16. $E_{[q]} := \{X_{q_1}(A, A) = \mathsf{x2e}([q_1], A, A)$    $q_1$ is a sub-query of $q\}$;
17. prune and return $E_{[q]}$;
```

Figure 9: Rewriting algorithm from qualifiers

of Q. To do so, it first computes $\mathsf{x2e}(p_i, B_j, B)$ for each (immediate) sub-query p_i of p at each possible DTD node B_j under A (i.e., B_j in reach (p, A)); then, it combines these $\mathsf{x2e}(p_i, B_j, B)$'s to get $\mathsf{x2e}(p, A, B)$. The details of this combination are determined based on the formation of p from its immediate sub-queries p_i, if any (cases 1-7). These cases are illustrated as follows.

First, when p is empty path ϵ, element type C, wildcard $*$ or descendants-or-self $//$, namely, cases (1)–(3) and (5), $\mathsf{x2e}(p, A, B)$ and $\mathsf{reach}(p, A)$ are determined by the DTD D alone regardless of the input query Q; thus it can be precomputed for each A, B, once and for all, and made available to XPathToEXp. We include these cases (cases (1)-(3)) in Fig. 8 for ease of reference (lines 9-14).

When $p = p_1/p_2$ (case (4)), for each C reached via p_1 from A, XPathToEXp assembles $\mathsf{x2e}(p_1, A, C)$ and $\mathsf{x2e}(p_2, C, B)$ for each B reached via p_2 from C to precisely represent paths from A to B in D. Furthermore, variables $X_{p_1}(A, C)$ and $X_{p_2}(C, B)$ are used instead of $\mathsf{x2e}(p_1, A, C)$ and $\mathsf{x2e}(p_2, C, B)$ to avoid that $\mathsf{x2e}(p, A, B)$ has an exponential size.

Similarly, in the case $p = \epsilon//p_1$ (case (5)), XPathToEXp assembles $\mathsf{x2e}(p_1, A, C)$ and $\mathsf{x2e}(p_2, C, B)$ for each C reached via $\epsilon//$ and each B reached via p_1 from C. Here $\mathsf{x2e}(\epsilon, A, C)$ is essentially $X[A, C, n]$, the variable in $\mathsf{rec}(A, C)$ representing the final result. Note that $\mathsf{rec}(A, C)$ is precomputed by Algorithm $\mathsf{CycleE_X}$, and is also added into E_Q in line 31.

When $p = p_1 \cup p_2$ (case (6)), for each B in D, XPathToEXp simply computes $\mathsf{x2e}(p_1, A, B) \cup \mathsf{x2e}(p_2, A, B)$. However, for the same reason given for case (4), $X_{p_1}(A, B)$ and $X_{p_2}(A, B)$ are used instead of $\mathsf{x2e}(p_1, A, B)$ and $\mathsf{x2e}(p_2, A, B)$

When p comes with a qualifier, i.e., when $p = p'[q]$ (case (case (7)), it invokes a procedure RewQual to translate $[q]$. Procedure RewQual may evaluate $[q]$ to a truth value (ϵ for *true* and $\emptyset$ for *false*) in certain cases based on the structure of the DTD D alone. If so, XPathToEXp simply drops $[q]$ in $\mathsf{x2e}(p, A, B)$, or leaves $\mathsf{x2e}(p, A, B)$ unchanged (i.e., $\emptyset$) if RewQual returns *false*.

At the end of the iteration, each $\mathsf{x2e}(p, A, B)$ is optimized by removing $\emptyset$, which returns an empty set over any XML tree, as described in Section 2. Finally, X_Q is defined to be the union of $\mathsf{x2e}(Q, r, B)$ for all $B \in \mathsf{reach}(Q, r)$, and the equations of the extended XPath query are put together into E_Q as the output of the algorithm (lines 29-32).

Procedure RewQual is shown in Fig. 9. It translates qualifiers $[q]$ into an extended XPath query, based on the structure of q. Furthermore, it "evaluates" q over the DTD and gets the truth value of $[q]$ if it can be determined based on the DTD structure alone. For example, when q is a path p, it concludes that $[q]$ is *false* if no node can be reached from A via p, and *true* if ϵ is contained in p, i.e., the current node is in the "result" of the query p. For Boolean operations, it invokes procedure optimize (not shown) that determines whether $q_1 \wedge q_2$, $q_1 \vee q_2$ and $\neg q_1$ can be evaluated to *true* or *false*. For $q_1 \wedge q_2$, for example, optimize evaluates it to *true* if both q_1 and q_2 are ϵ, and to *false* if one of q_1 and q_2 is $\emptyset$; similarly for $q_1 \vee q_2$ and $\neg q_1$.

Example 4.3: Recall the XPath query Q_2 from Example 2.2. Observe that the algorithm of [27] *cannot* handle this query over the *dept* DTD of Fig. 1 (a). In contrast, XPathToEXp translates Q_2 to the extended XPath query E_{Q_2} below:

$$\begin{aligned} X_{Q_2} = \ & dept/course[X_{course\_course}/prereq/course[cno="cs66"] \wedge \\ & \neg\ X_{course\_project} \wedge \neg\ takenBy/student/X_{qualified\_course}[cno="cs66"]] \end{aligned}$$

Let i_c, j_p, and k_q be the number assigned to *course*, *project*, and *quantified*, respectively, and let n be the number of nodes in the DTD of Example 2.2. Here, $X_{\text{course\_course}} = X[i_c, i_c, n]$, obtained by computing $\mathsf{rec}(course, course)$; $X_{\text{course\_project}} = X[i_c, j_p, n]$, obtained by computing $\mathsf{rec}(course, project)$; and $X_{\text{qualified\_course}} = X[k_q, i_c, n]$, obtained by computing $\mathsf{rec}(qualified, course)$.

The algorithm of Section 5 can then translate E_{Q_2} to equivalent relational queries. □

The result below tells us that Algorithm XPathToEXp computes extended XPath queries in low polynomial time, as desired.

Theorem 4.2: *Each* XP*ath query Q over a* DTD *D can be rewritten to an extended* XP*ath expression E_Q in $O(|D|^3 * log|D| * |Q| * log|Q|)$ time, such that E_Q is equivalent to Q over all* DTD*s that contain D, and that the size of E_Q is bounded by $O(|D|^3 * |Q| * log|D|)$.* □

Proof: For the complexity, observe the following. (a) Algorithm XPathToEXp produces an extended XPath query with $O(|D|^2 * |Q|)$ many variables and $O(|D|^2 * |Q|)$ many equations, each consisting of at most two variables and thus takes $O(log(|D||Q|)$ space (to encode the variables). (b) In addition, $\mathsf{rec}(A, B)$ for '//' computed by Algorithm contains $O(|D|^3)$ many equations, which takes $O(|D|^3 * log|D|)$ space. Putting these together, the extended XPath query produced takes no more than $O(|D|^3 * log|D| * |Q|)$ space. (c) Each step of the inner-most loop of Algorithm XPathToEXp takes at most $O(|D|^2 * log|Q| * log|D|)$ time, where $log|Q| * log|D|$ is for writing the variables involved. Thus the algorithm takes no more than $O(|D|^3 * log|D| * |Q| * log|D|)$ time.

We show that E_Q is equivalent to Q over all DTDs that contain D, by induction on the structure of Q. For the base cases, i.e., when Q is ϵ, A, $*$ and $//$, the statement trivially holds. In particular, the argument for // is given in the proof of Theorem 4.1.

For the inductive step, assume that the statement holds for sub-queries p_1, p_2 of Q, i.e., $\mathsf{x2e}(p_i, A, B)$ is equivalent to p_i w.r.t. A and B for all $A, B \in D$ and $i = 1, 2$. We show that the statement holds for p_1/p_2. Proofs for the other cases are similar.

Let D' be an arbitrary DTD that contains D, and σ be the mapping that, given any instance T of D', extracts a subtree V of T that is an instance of D, as specified in Section 3. Let v be an A element in V, where σ maps v to an A element $\sigma(v)$ in T. We need to show that a node u' can be reached from $\sigma(v)$ via $\mathsf{x2e}(p_1/p_2, A, B)$ in T iff there exists a node u in V such that $u' = \sigma(u)$ and u can be reached from v via p_1/p_2 in V.

First, for any node u in V reached from v via p_1/p_2, there must be a node w in V such that w is reached from v via p_1 and u is reached from w via p_2. Assume w is labeled C. Then obviously $C \in \mathsf{reach}(p_1, A)$. By the induction hypothesis and the definition of σ, there exist w', u' in T such that $w' = \sigma(w)$, $u' = \sigma(u)$; moreover, w' can be reached from $\sigma(v)$ via $\mathsf{x2e}(p_1, A, C)$ and u' can be reached from w' via $\mathsf{x2e}(p_2, C, B)$. Thus from the definition of $\mathsf{x2e}(p_1/p_2, A, B)$ and the semantics of XPath, it follows that u' can be reached from $\sigma(v)$ via $\mathsf{x2e}(p_1/p_2, A, B)$ in T.

Conversely, for any node u' in T reached from $\sigma(v)$ via $\mathsf{x2e}(p_1/p_2, A, B)$, by the definition of $\mathsf{x2e}(p_1/p_2, A, B)$ and the semantics of XPath, there must be a node w' in T such that w is reached from $\sigma(v)$ via $\mathsf{x2e}(p_1, A, C)$ and u' is reached from w' via $\mathsf{x2e}(p_2, C, A)$, where w' is labeled C. By the induction hypothesis for p_1, there exists w in V such that $w' = \sigma(w)$, and w can be reached from v via p_1. Moreover, by the definition of σ, $C \in \mathsf{reach}(p_1, A)$. Then by the induction hypothesis for p_2, there exists u in V such that $u' = \sigma(u)$ and u can be reached from w via p_2. By the semantics of XPath, it follows that u can be reached from v via p_1/p_2 in V. □

Observe the following. First, extended XPath queries capture DTD *recursion and* XP*ath recursion* in a uniform framework by means of the general Kleene closure E^*. The use of variables makes it possible to translate XPath queries in polynomial time, in contrast to the exponential blowup of query translation into regular XPath [15]. Second, during the translation, algorithm XPathToEXp conducts *optimization leveraging the structure of the* DTD. Third, Kleene closure is only introduced when computing $\mathsf{rec}(A, B)$; thus there are no qualifiers *within* a Kleene closure E^* in the output extended query. Fourth, both $|Q|$ and $|D|$ are far smaller than the data (XML tree) size in practice. Finally, as remarked earlier, Algorithm XPathToEXp also provides query answering ability for XPath queries over certain virtual XML views.

5 From Extended XPath Expressions to SQL

In this section we present an algorithm, Algorithm EXpToSQL, for rewriting extended XPath expressions into equivalent SQL queries with the simple LFP operator. Together with Algorithm XPathToEXp given in the last section, these provide a solution for answering XPath queries on XML data stored in relations. We also discuss optimization of the produced SQL queries.

5.1 Translation Algorithm

Consider a mapping $\tau : D \to \mathcal{R}$, where D is a DTD and $\mathcal{R}$ is a relational schema, such that its associated data mapping τ_d shreds XML trees of D into databases of $\mathcal{R}$. Given an extended XPath expression E_Q over D, Algorithm EXpToSQL computes a sequence Q' of equivalent relational queries with the simple LFP operator Φ such that for any XML tree T of D, $E_Q(T) = Q'(\tau_d(T))$. We write Q' in the relational algebra (RA), which can be easily coded in SQL.

More specifically, Q' is a list of the form $R_e \leftarrow \mathsf{e2s}(e)$, where e is an sub-expression of the extended XPath expression E_Q, R_e is a temporary table which is used in later queries, and $\mathsf{e2s}(e)$ is the RA query equivalent to e. The list Q' is properly ordered such that if e is a sub-expression of e', then $R_e \leftarrow \mathsf{e2s}(e)$ precedes $R_{e'} \leftarrow \mathsf{e2s}(e')$ in Q', i.e., when $\mathsf{e2s}(e)$ is needed, the query has already been evaluated and its result is available in R_e, which can be directly used in $\mathsf{e2s}(e')$.

Algorithm EXpToSQL suffices to translates the query produced by Algorithm XPathToEXp into equivalent SQL queries. To see this, observe the following. First, the equations in the extended XPath query returned by Algorithm XPathToEXp can be sorted as $(X_{p_1} = E_{p_1}, \ldots, X_{p_m} = E_{p_m})$, where p_i's are sub-queries of the input query Q such that all sub-queries of a sub-query p precede p, with $p_m = Q$. In other words, X_{p_i} only appears in E_{p_j} if $i < j$. We can apply Algorithm EXpToSQL to $E_{p_1}, \ldots, E_{p_m}$ one by one in this order, creating a temporary table R_{p_i} for each $\mathsf{e2s}(E_{p_i})$. For each occurrence of X_{p_i} in E_{p_j}, we simply use R_{p_i} in $\mathsf{e2s}(E_{p_j})$. Thus it suffices for EXpToSQL to translate extended XPath expressions E_{p_i}. Second, the query returned by XPathToEXp is equivalent to the XPath query Q over all DTDs that contain D. Thus it is equivalent to Q over D since D contains itself. Putting these together, we have that EXpToSQL and XPathToEXp translate the XPath Q over D to equivalent SQL queries over $\mathcal{R}$.

We show Algorithm EXpToSQL in Fig. 10. The algorithm takes an extended XPath expression E_Q over the DTD D as input, and returns an equivalent sequence Q' of RA queries with the LFP operator Φ as output. The algorithm is based on dynamic programming: for each sub-expression e of E_Q, it computes $\mathsf{e2s}(e)$, which is the RA query translation of e; it then associates $\mathsf{e2s}(e)$ with a temporary table R_e and increments the list Q' with $R_e \leftarrow \mathsf{e2s}(e)$. More specifically, $\mathsf{e2s}(e)$ is computed by assembling $\mathsf{e2s}(e_i)$ where e_i's are its immediate sub-queries. Thus upon the completion of the processing one will get the list Q' equivalent to E_Q. To do this, the algorithm first finds the list L of all sub-expressions of E_Q and topologically sorts them in ascending order (line 1). Then, for each sub-query e in L, it computes $\mathsf{e2s}(e)$ (lines 3–24), bottom-up starting from the inner-most sub-query of E_Q, and based on the structure of e (cases (1)-(12)).

As subtle issue is that the LFP operator Φ supports $(E)^+$ but not $(E)^*$ (where $(E)^*$ means repeating E zero or more times, while $(E)^+$ indicates repeating E at least once). Thus $(E)^*$ needs to be converted to $\epsilon \cup (E)^+$. To simplify the handling of ϵ, we assume a relation R_{id} consisting of tuples $(v, v, v.val)$ for all nodes (IDs) v in the input XML tree except the root r. Note that R_{id} is the identity relation for join operation: $R \bowtie R_{id} = R_{id} \bowtie R = R$ for any relation R. With this we translate $(E)^*$ to $\Phi(R) \cup R_{id}$, where R codes E, and R_{id} tuples will be eliminated in a later stage. We rewrite ϵ into R_{id} just to simplify the presentation of our algorithm; a more efficient translation is adopted in our implementation.

More specifically, EXpToSQL handles different cases of e as follows.

(Case 2) It rewrites a label A to the corresponding relation R_A.

(Case 3) It replaces each occurrence of variable X with its corresponding temporary table R_X. From the discussion above one can see that for each X appearing in an expression E_{p_j}, $X = E_{p_i}$ has already been processed and a table R_X has been associated with $\mathsf{e2s}(E_{p_i})$.

(Case 4) Concatenation is coded with projection Π and join $\bowtie$.

(Cases 5, 11) Union and disjunction are encoded with union $\cup$ in relational algebra.

(Case 6) Kleene closure $(E)^*$ is converted to the LFP operator Φ, as remarked above.

```
Algorithm EXpToSQL
Input: an extended XPath expression $E_Q$ over a DTD $D$.
Output: an equivalent list $Q'$ of RA queries over $\mathcal{R}$, where $\tau : D \to \mathcal{R}$.
1.  compute the ascending list $L$ of sub-expressions in $E$;
2.  $Q'$ := empty list [ ];
3.  for each $e$ in the order of $L$ do
4.      case $e$ of
5.      (1) $\epsilon$:  $\mathsf{e2s}(e) := R_{id}$;
6.      (2) $A$:  $\mathsf{e2s}(e) := R_A$;
7.      (3) $X$:  $\mathsf{e2s}(e) := R_X$;   /* $R_X$ is a temporary table for storing $E$ where $X = E$ */
8.      (4) $e_1/e_2$:  let $R_1 = \mathsf{e2s}(e_1)$, $R_2 = \mathsf{e2s}(e_2)$;
9.                  $\mathsf{e2s}(e) := \Pi_{R_1.F, R_2.T, R_2.V}(R_1 \bowtie_{R_1.T = R_2.F} R_2)$;
10.     (5) $e_1 \cup e_2$:  let $R_1 = \mathsf{e2s}(e_1)$, $R_2 = \mathsf{e2s}(e_2)$;
11.                 $\mathsf{e2s}(e) := R_1 \cup R_2$;
12.     (6) $E^*$:  let $R = \mathsf{e2s}(e)$;
13.                 $\mathsf{e2s}(e) := \Phi(R) \cup R_{id}$;
14.     (7) $e_1[q]$:  let $R_1 = \mathsf{e2s}(e_1)$, $R_q = \mathsf{e2s}(q)$;
15.         $\mathsf{e2s}(e) := \Pi_{R_1.F, R_1.T, R_2.V}(R_1 \bowtie_{R_1.T = R_q.F} R_q)$;
                    /* returns $R_1$ tuples that connect with $R_2$ tuples */
16.     (8) $[e_1]$:  $\mathsf{e2s}(e) := \mathsf{e2s}(e_1)$;
17.     (9) $e_1[text() = c]$:  let $R_1 = \mathsf{e2s}(e_1)$;
18.                 $\mathsf{e2s}(e) := \sigma_{R_1.V = c} R_1$;   /* select tuples $t$ of $R_1$ with $t.V = c$ */
19.     (10) $[q_1 \wedge q_2]$:  let $R_1 = \mathsf{e2s}(q_1)$;  $R_2 = \mathsf{e2s}(q_2)$;
20.                 $\mathsf{e2s}(e) := R_1 \cup R_2 \setminus ((R_1 \setminus R_2) \cup (R_2 \setminus R_1))$;   /* $\mathsf{e2s}(e) = R_1 \cap R_2$; */
21.     (11) $[q_1 \vee q_2]$:  let $R_1 = \mathsf{e2s}(q_1)$;  $R_2 = \mathsf{e2s}(q_2)$;
22.                 $\mathsf{e2s}(e) := R_1 \cup R_2$;
23.     (12) $e_1[\neg q]$:  let $R_q = \mathsf{e2s}(q)$,  $R_1 = \mathsf{e2s}(e_1)$;
24.                 $\mathsf{e2s}(e) := R_1 \setminus \Pi_{R_1.F, R_1.T, R_1.V}(R_1 \bowtie_{R_1.T = R_q.F} R_q)$;
                    /* only $R_1$ tuples not connecting to any $R_q$ tuple */
25.     $Q' := (R_e \leftarrow \mathsf{e2s}(e)) :: Q'$;   /* add $\mathsf{e2s}(e)$ to $Q'$ */
26. $\mathsf{e2s}(E_Q) := \sigma_{F='\_'}\mathsf{e2s}(E_Q)$;  /* select nodes reachable from root */
27. $Q' := \mathsf{e2s}(E_Q) :: Q'$;
28. optimize $Q'$ by extracting common sub-queries;
29. return $Q'$;
```

Figure 10: Rewriting algorithm from extended XPath to SQL

(Case 10) Conjunction is coded with set intersection implemented with union $\cup$ and set difference $\setminus$ in relational algebra.

(Qualifiers) An expression with qualifier $e = e_1[q]$ is converted to a RA query $\mathsf{e2s}(e)$ that returns only those $\mathsf{e2s}(e_1)$ tuples t_1 for which there exists a $\mathsf{e2s}(q)$ tuple t_2 with $t_1.T = t_2.F$, i.e., when the qualifier q is satisfied at the node represented by $t_1.T$ (case 6). For example, it converts $[e_1]$ to $\mathsf{e2s}(e_1)$ when e_1 is an extended XPath expression (case 7). There are, however, two special cases. First, it rewrites $e_1[\neg q]$ to a RA query $\mathsf{e2s}(e)$ that returns only those $\mathsf{e2s}(e_1)$ tuples t_1 for which there exists no $\mathsf{e2s}(q)$ tuple t_2 such that $t_1.T = t_2.F$, i.e., when the qualifier q is not satisfied at the node $t_1.T$ (and hence $[\neg q]$ is satisfied at $t_1.T$ (case 11)); this captures precisely the semantics of negation in XPath. Second, it rewrites $e = e_1[text() = c]$ in terms of selection σ that returns all tuples of $\mathsf{e2s}(e_1)$ that have the text value c.

F	T
d_1	c_1
c_1	c_2
c_2	c_3
p_1	c_4
s_2	c_5
c_1	c_5
c_2	c_4

(a) R

F	T
c_1	c_2
c_1	c_3
c_1	c_4
c_1	c_5
$\cdots$	$\cdots$

(b) R_γ

F	T
d_1	p_1
d_1	p_2

(c) R_f

Table 3: Intermediate and final results of `dept//project`.

In each of the cases above, the list Q' is incremented by adding $R_e \leftarrow \mathsf{e2s}(e)$ as the head of Q' (line 25). After the iteration it yields $\sigma_{F='\_'}\mathsf{e2s}(E_Q)$ (line 26), which selects only those nodes reachable from the root of the XML tree, removing unreachable nodes including those introduced by R_{id}. It also optimizes the sequence Q' of RA queries by eliminating empty set and extracting common sub-queries (details omitted from Fig. 10), and returns the cleaned Q' (lines 28–29).

Example 5.1: Consider the XPath query $Q_1 =$ `dept//project` over the `dept` DTD of Fig. 1 (a). Over the simplified DTD is Fig. 1 (b), Q_1 becomes $R_d//R_p$. A possible equivalent RA translations Q'_1 has been given in Example 3.2, which includes a single LFP operation $R_\gamma = \Phi(R) \cup \Pi_{T,T}(R_c)$, where $R = R_{cc} \cup R_{csc} \cup R_{cpc}$. When evaluated over the relational database of Fig. 1 (which encodes an XML tree of the `dept` DTD), Q'_1 produces R, R_γ, and the final result as shown in Table 3 (a), (b) and (c), respectively.

As another example, recall the XPath query Q_2 from Example 2.2, and its extended XPath query translation X_{Q_2} from Example 4.3, which contains $X_{\text{course\_course}}$ $(= X[i_c, i_c, n])$, $X_{\text{course\_project}}$ $(= X[i_c, j_p, n])$, and $X_{\text{qualified\_course}}$ $(= X[k_q, i_c, n])$ computed by Algorithm $\mathsf{CycleE_X}$. Given X_{Q_2}, the $\mathsf{EXpToSQL}$ algorithm generates the RA translation below:

$$X_{\text{course\_course}}/prereq/course[cno ='' cs66"] \;:\; \sigma_{cno=``cs66"}(R_{cc} \bowtie R_c)$$
$$takenBy/student/X_{\text{qualified\_course}}[cno ='' cs66"] \;:\; \sigma_{cno=''cs66"}(R_s \bowtie R_{qc})$$

Suppose the results of $X_{\text{course\_course}}$, $X_{\text{course\_project}}$, and $X_{\text{qualified\_course}}$ are stored in R_{cc}, R_{cp}, and R_{qc}, respectively.

Note that Q_2 is of the form (with a complex qualifier) $dept/course[q_1 \wedge \neg q_2 \wedge \neg q_3]$, which is handled by our algorithms by treating it as $Q_2^1 = dept/course[q_1]$, $Q_2^2 = Q_2^1[\neg q_2]$ and $Q_2 = Q_2^2[\neg q_3]$. Therefore, $Q_2^1 \leftarrow R_d \bowtie R_c \bowtie R_1$, $Q_2^2 \leftarrow Q_2^1 \setminus (Q_2^1 \bowtie R_{cp})$, and X_{Q_2} becomes $Q_2^2 \setminus (Q_2^2 \bowtie R_2)$ where projections are omitted. In contrast, the algorithm of [27] cannot translate XPath queries of this form to relational queries. $\square$

The corollary below confirms that our translation algorithms provide an effective solution for answering XPath queries over (possibly recursive) DTDs by means of traditional RDBMS.

Corollary 5.1: *Each XPath query Q over a DTD D can be rewritten to an equivalent sequence of SQL queries (with the LFP operator) in $O(|D|^3 * log|D| * |Q| * log|Q|)$ time.* $\square$

Proof: It is easy to verify the following. (a) Given an input extended XPath expression E_Q, Algorithm $\mathsf{EXpToSQL}$ takes $O(|E_Q|)$ time to compute a sequence Q' of SQL queries (with the LFP operator). The size of Q' is also $O(|E_Q|)$. (b) The SQL queries Q' are equivalent to E_Q, i.e., for any XML tree T of D, $E_Q(T) = Q'(\tau_d(T))$. This can be verified by induction on the structure of E_Q. Recall from Theorem 4.2 that given an input XPath query Q over D, Algorithm XPathToEXp computes an extended XPath query E_Q equivalent to Q over all DTDs that contain D. The query E_Q can be computed in $O(|D|^3 * log|D| * |Q| * log|Q|)$ time, and its size is in $O(|D|^3 * |Q| * log|D|)$. Putting these together, we have that the SQL queries Q' can be computed from Q by using XPathToEXp followed by EXpToSQL in $O(|D|^3 * log|D| * |Q| * log|Q|)$ time. Furthermore, the size of Q' is in $O(|D|^3 * |Q| * log|D|)$. □

Observe the following. First, algorithm EXpToSQL shows that the simple LFP operator $\Phi(R)$ suffices to express XPath queries over recursive DTDs; thus there is no need for the advanced SQL'99 recursion operator. Second, the total size of the produced SQL queries is bounded by a low polynomial of the sizes of the input XPath query Q and the DTD D. Finally, the algorithms XPathToEXp and EXpToSQL can be easily combined into one; we present them separately to focus on their different functionality.

5.2 Optimization: Pushing Selections into the LFP Operator

Algorithms XPathToEXp and EXpToSQL show that SQL with the simple LFP operator is powerful enough to answer XPath queries over recursive DTDs. While certain optimizations are already conducted during the translation, other techniques, e.g., sophisticated methods for pushing selections/projections into the LFP operator [2, 1, 5, 4, 6] can be incorporated into our translation algorithms to further optimize generated relational queries.

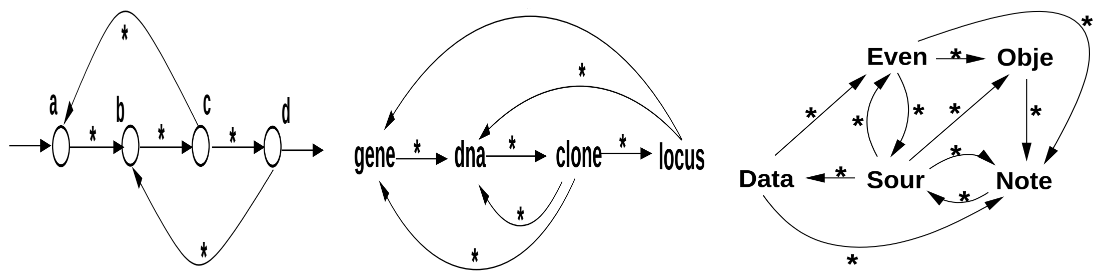
We next show how to push selections into LFP. Consider an XPath query $Q_3 = R_d[id = a]/R_c//R_p$. To simplify the discussion, assume that our algorithms rewrite Q_3 into $R_1 \leftarrow Q_d$ and $R_2 \leftarrow \text{LFP}(R_0)$, where Q_d and $\text{LFP}(R_0)$ compute $R_d[id = a]$ and $R_c//R_p$, (i.e., $\text{rec}(R_c, R_p)$), respectively. While $R_1 \bowtie R_2$ yields the right answer, we can improve the performance by pushing the selection into the LFP computation such that it only traverses "paths" starting from the R_c children of those R_d nodes with $id = a$. Recall from Eq. (2) that one can specify a predicate C on the join between R_Φ and R_0 in LFP, where R_0 is the input relation and R_Φ is the relation being computed by the LFP (Section 3; supported by *connectby* of Oracle and *with...recursion* of IBM DB2). That is, $R^0_\Phi \leftarrow R_0$, $R^i_\Phi \leftarrow R^{i-1}_\Phi \cup (R^{i-1}_\Phi \bowtie_C R_0)$, and finally, $R_\Phi \leftarrow R^m_\Phi$, where $R^m_\Phi = R^{m+1}_\Phi$, i.e., the fixpoint. Here $C = R_\Phi.F \in \pi_T(R_1) \wedge R^{i-1}_\Phi.T = R_0.F$. ('$\in$' denotes *in* in SQL), i.e., besides the equijoin $R_\Phi.T = R_0.F$ we want the F (*from*) attribute of R_Φ to match the T (*to*) attribute of R_1. Then, each iteration of the LFP only adds tuples (f, t), where f is a child of a node in $\pi_T(R_1)$.

Similarly the selection in $R_d//R_c/R_p[id{=}c]$ can be pushed into $\text{LFP}(R_0)$ for $\text{rec}(R_d, R_c)$. Let R_1 be the relation found and the LFP join condition be: $R^{i-1}_\Phi.F = R_0.T \wedge R_\Phi.T \in \pi_F(R_1)$. Then the LFP only returns tuples of the form (f, t), where t is the parent of a node in $\pi_F(R_1)$.

In general, given an extended XPath expression $R_1 \bowtie \text{LFP}(R_0)$ (or $\text{LFP}(R_0) \bowtie R_1$), where R_1 is associated with a selection condition, we can push the selection into LFP along the same lines as above. Let us denote the query resulted from this as $\text{push}(R_1, R_0)$.

Below we identify general cases where the push operation can be applied. We may decompose a list $\bar{R}$ of RA queries and employ the push operation as follows:

(i) by union: $\bar{R} = R_1 \bowtie \text{LFP}(R_0) \cup R'_1 \bowtie \text{LFP}(R_0)$, and we can rewrite $\bar{R}$ to equivalent yet more efficient $\bar{R} = \text{push}(R_1, R_0) \cup \text{push}(R'_1, R'_0)$;



(a) Cross (b) BIOML (c) GedML

Figure 11: DTD Graphs

(ii) by conjunction: $\bar{R} = R_1 \bowtie \text{LFP}(R_0) \bowtie R_1' \bowtie \text{LFP}(R_0)$; in this case we can rewrite $\bar{R}$ to $\bar{R} = \mathsf{push}(\mathsf{push}(R_1, R_0) \bowtie R_1', R_0')$;

(iii) by nest: $\bar{R} = R_2 \bowtie \text{LFP}(R_1 \bowtie \text{LFP}(R_0))$, and we can rewrite it to $\bar{R} = \mathsf{push}(R_2, \mathsf{push}(R_1, R_0))$.

In particular, $\bar{R} = R_2 \bowtie R_1 \bowtie \text{LFP}(R_0)$ and $\bar{R} = R_2 \bowtie \text{LFP}(R_1 \bowtie R_0)$ can be optimized following cases (ii) and (iii) above. Here we assume that there may exist arbitrary selection condition on R_1 or R_1'. In fact, we can push selections into LFP even when there are no explicit user-given selection conditions. Consider, for example, $R_{1_1} \bowtie R_{1_2} \bowtie \cdots \bowtie R_{1_n} \bowtie \text{LFP}(R_0)$. By first computing joins $R_{1_1} \bowtie R_{1_2} \bowtie \cdots \bowtie R_{1_n}$ followed by projection on R_{1_n}, namely, $\pi_{R_{1_n}}(R_{1_1} \bowtie R_{1_2} \bowtie \cdots \bowtie R_{1_n})$, we can also push this query into $\text{LFP}(R_0)$ and thus speed up the computation of the fixpoint. As will be seen in Section 6, this optimization is effective.

Other Optimization: Besides the push operation, several other optimization techniques can be used to improve the rewritten SQL queries. Observe that in our generated relational queries, all joins and unions are outside of the LFP operator, as opposed to embedding joins/unions in the blackbox of the operator *with...recursive*. As a result, one can capitalize on RDBMS to optimize those joins/unions. Indeed, making use of relational optimizers is one of the reasons for one to want to push the work to RDBMS before XML query optimizers become as sophisticated as their RDBMS counterparts. Furthermore, our translation framework makes it easy to accommodate all existing techniques in commercial RDBMS [36, 21]; in particular, multi-query optimization techniques (e.g., [39]) can be easily incorporated into our framework to optimize a sequence of SQL queries produced by our algorithms.

XML Reconstruction: It is worth mentioning that our rewriting algorithms can be easily extended such that they not only find ancestor/descendant pairs, but also preserve the path information between each pair. A simple way to do so is to use an additional attribute P in LFP $\Phi()$ such that the P attribute keeps track of the path information by concatenating edges when tuples are joined. Both DB2 and Oracle support such a string concatenation operator.

6 A Performance Study

To verify the effectiveness of our rewriting and optimization algorithms, we have conducted a performance study on evaluating XPath queries using an RDBMS with three approaches:

- the SQLGen-R algorithm proposed in [27],

- our rewriting algorithms by using Tarjan's method (referred to as CycleE of Fig. 6) to find rec(A, B), i.e., paths from node A to B in a DTD graph, and
- our rewriting algorithms by using CycleE$_X$ of Fig. 7 to compute rec(A, B).

We experimented with these algorithms using a simple yet representative DTD and two complex DTDs from real world. The simple DTD is depicted in Fig. 11 (a) (2 cross cycles). The two real-life DTDs are (1) a 4-cycle DTD extracted from **BIOML** (BIOpolymer Markup Language [7]), as shown in Fig. 11 (b); and (2) a 9-cycle DTD extracted from **GedML** (Genealogy Markup Language [20]), given in Fig. 11 (c).

While testing several different types of XPath queries, our performance study focuses on the evaluation of // because // is the only operator in XPath queries that, in the presence of recursive DTDs, leads to Kleene closures and therefore LFP in RDBMS, and is a dominant factor of XPath query evaluation. Two considerations on query evaluation are given below. First, as shown in our rewriting algorithms, // is translated into a sequence of projection, join and union, along with LFP. The evaluation of this sequence should be isolated from other operators that do not contribute to the evaluation of //. Second, the non-recursive operators in XPath queries are translated into selection, projection, join and union that the existing relational query processing techniques can support, and is beyond the scope of this evaluation.

Our experimental results demonstrate that our rewriting algorithms with CycleE$_X$ outperform the other two in most cases.

Implementation. We have implemented a prototype system supporting SQLGen-R, CycleE and CycleE$_X$, using Visual C++, denoted by **R**, **E** and **X**, respectively, in all the figures. SQLGen-R rewrites a query with the *with...recursive* operator, while CycleE and CycleE$_X$ translate a query to a sequence of SQL queries. We run a batch to execute these rewritten SQL queries. We conducted experiments using IBM DB2 Enterprise 9 on a single 2.8GHz CPU with 1GB main memory. We did not compare SQLGen-R with ours on Oracle, because Oracle does not support the SQL'99 recursion. The queries output ancestor-descendant pairs.

Testing Data: Testing data were generated using IBM XML Generator (`http://www.alphaworks.ibm.com`). The input to the IBM XML Generator is a DTD file and a set of parameters. We mainly control two parameters, X_L and X_R, in order to study the impacts of the shape of XML trees. Here X_L is the maximum number of levels in the resulting XML tree. If a tree goes beyond X_L levels, it will add none of the optional elements (denoted by * or ? in the DTD) and only one of each of the required elements (denoted by + or with no option); X_R controls the maximum number of occurrences of child elements in the presence of the $*$ or $+$ option. In other words, the number of children of each element of a type defined with this option is a random number between 0 and X_R. Together X_L and X_R determine the shape of an XML tree: the larger the X_L value, the deeper the generated XML tree; and the larger the X_R value, the wider the XML tree. The default values used in our testing for X_L and X_R are 4 and 12, respectively. The default number of elements in a generated XML tree is 120,000. There is a need to control the sizes of XML trees to be the same in different settings for comparison purposes, and thus excessively large XML trees generated were trimmed. For the other parameters of the Generator, we used their default settings.

Relational Database. The generated XML data was mapped to a relational database using the shared-inlining technique [43]. Indexes were generated for all possible joined attributes.

Experimental Study. We conducted five sets of experiments. (1) We tested four XPath queries: a query with //, a twig join query, a query with $\neg$ and //, and a query with $\neg$, $\vee$, $\wedge$ and //. The testing was done using different databases (fixing the database size while varying the relation sizes). (2) In the second set of experiments we evaluated the effectiveness of our optimization method by pushing selections into the LFP operator. (3) We tested the scalability of our generated SQL queries w.r.t. different database sizes using a query containing //. Experiments (1) – (3) were conducted with the simple cross-cycle DTD graph. (4) We tested several XPath queries with different DTDs, which are subgraphs of the real-life DTDs BIOML using the same database. The main difference between (1) and (4) is that the former tested the same queries with different databases, and the latter tested different queries with the same database. (5) Finally, we examined the numbers of operators (LFP, etc) in the SQL queries generated by CycleE and CycleE$_X$, respectively.

6.1 Exp-1: Evaluation of Selective Queries

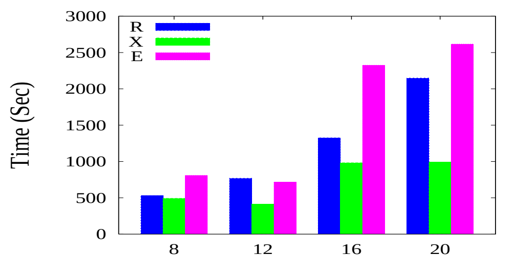
In this study, over the simple cross-cycle DTD (Fig. 11 (a)), we tested the following four XPath queries:

- Q_a = `a/b//c/d` (with //),
- Q_b = `a[//c]//d` (a twig join query),
- Q_c = `a[¬ //c]` (with $\neg$ and //), and
- Q_d = `a[¬ //c ∨ (b ∧ //d)]` (with $\neg$, $\vee$, $\wedge$ and //).

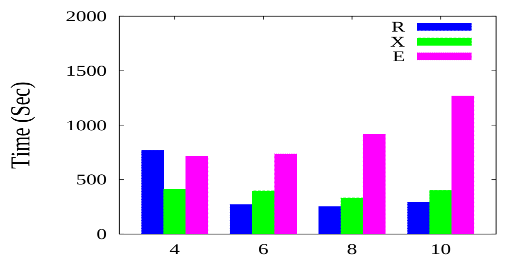
The XPathToEXp algorithm translates these XPath queries into four extended XPath expressions, namely, $Q'_a = a/X_{\text{b\_c}}/d$, $Q'_b = a[X_{\text{a\_b}}/c]/X_{\text{a\_c}}/d$, $Q'_c = a[\neg X_{\text{a\_b}}/c]$, and $Q'_d = a[\neg X_{\text{a\_b}}/c \vee (b \wedge X_{\text{a\_c}}/d)]$, respectively. Here, $X_{\text{b\_c}}$, $X_{\text{a\_b}}$, and $X_{\text{a\_c}}$ will be computed by $\mathsf{rec}(b, c)$, $\mathsf{rec}(a, b)$, and $\mathsf{rec}(a, c)$ using CycleE and CycleE$_X$ to test CycleE and CycleE$_X$, respectively. Since the last three XPath queries cannot be handled by SQLGen-R, we tested SQLGen-R by generating a *with...recursive* query for each $\mathsf{rec}(A, B)$ in our translation framework.

We used an XML tree with a fixed size of 120,000 elements. The same queries were evaluated over different shapes of XML trees controlled by the height of the tree (X_L) and the width of tree (X_R). Since an XML tree with different heights and/or widths results in different sizes of relations in a database, even though the database size is the same, the same translated SQL query may end up having different query-processing costs. We report elapsed time (seconds) for each query in Fig. 12. For a single query, one figure shows the elapsed time while varying X_L from 8 to 20 with $X_R = 4$, whereas the other figure shows the elapsed time while varying X_R from 4 to 10 with $X_L = 12$.

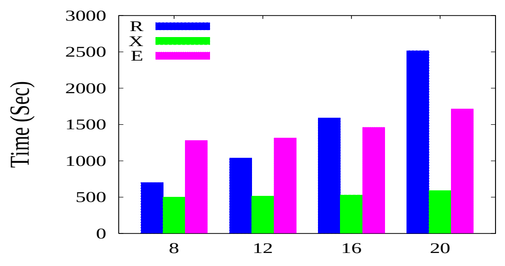
Fig. 12 (a), (c), (e), and (g), show the elapsed time while varying X_L, when X_R is fixed. The XML trees become higher, but the distribution of widths in the XML trees remains unchanged, while X_L increases. The elapsed time for all the three approaches increases. As can be seen from the figures, the performance of SQLGen-R and CycleE is significantly affected while X_L increases. However, the performance CycleE$_X$ is marginally affected. CycleE$_X$ noticeably outperforms SQLGen-R and CycleE.



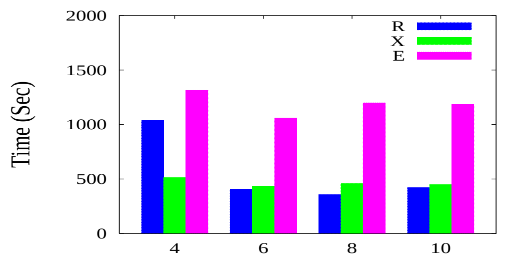
(a) Q_a: Vary X_L



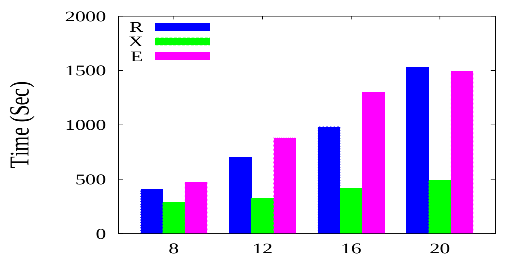
(b) Q_a: Vary X_R



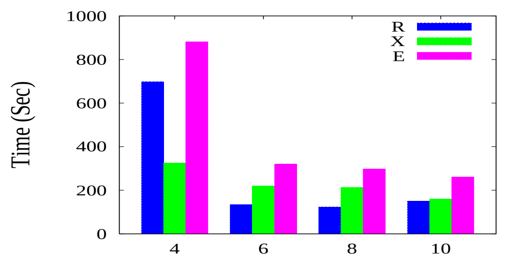
(c) Q_b: Vary X_L



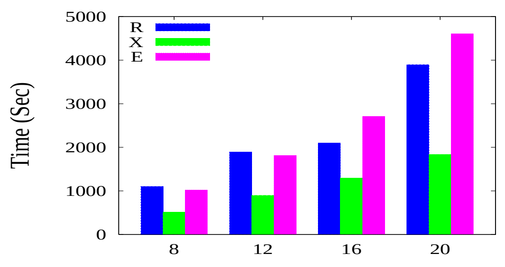
(d) Q_b: Vary X_R



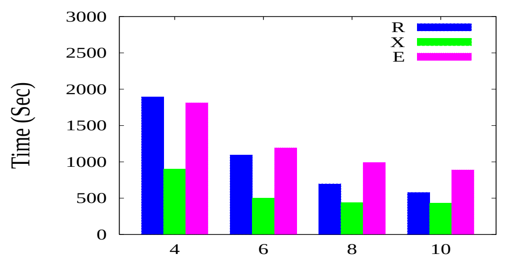
(e) Q_c: Vary X_L



(f) Q_c: Vary X_R



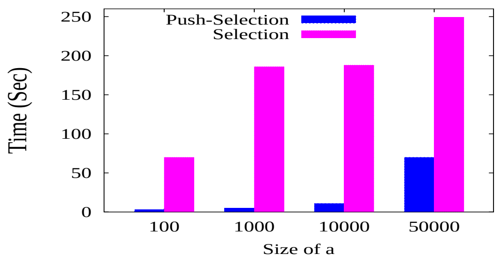
(g) Q_d: Vary X_L



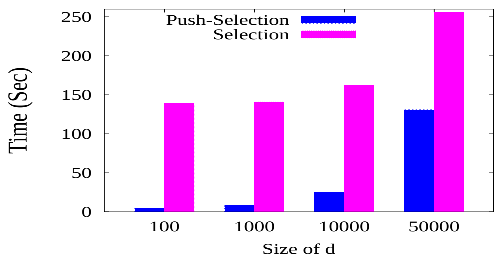
(h) Q_d: Vary X_R

Figure 12: Processing time for cross cycles (Fig. 11 (a)).

Fig. 12 (b), (d), (f), and (h), show the elapsed time while varying X_R, when X_L is fixed. In other words, the average number of children per element in an XML increases, and the height of the XML tree remains unchanged, while X_R increases. More precisely, the XML generator generates an XML tree with more elements at the leaf level for a larger X_R value. The percentages of the leaf nodes in the XML trees are 50%, 67%, 74%, 80%, when $X_R = 4$, $X_R = 6$, $X_R = 8$, and $X_R = 10$, respectively. With such distributions, SQLGen-R performs better, while X_R increases. It is difficult to analyze the *with...recursive*, but it can be because the most results are computed in a few iterations. CycleE$_X$ is marginally affected by the changes of X_R values, it shows similar performance while X_R increases. CycleE performs worst due to the large number of operations it needs to perform.



(a) Vary the Number of a Elements



(b) Vary the Number of d Elements

Figure 13: Pushing Selection ($X_R = 8$ and $X_L = 12$)

6.2 Exp-2: Pushing Selections into LFP

We tested two XPath queries with selection conditions: $Q_e = a[id = A_i]/b//c/d$ and $Q_f = a/b//c/d[id = D_i]$. For each query we generated two SQL queries, one with selections pushed into LFP and the other without. We evaluated these queries using datasets of the DTD of Fig. 11 (a), fixing the size of the datasets while varying the size of the set selected by the qualifiers of A_i and D_i. Figures 13(a) and 13(b) show the results. In Fig. 13(a), we vary the number of qualified a elements from 100 to 50,000, while in Fig. 13(b), we vary the number of qualified d elements from 100 to 50,000. It is shown that as expected, performance improvement by pushing selections into the LFP operator is significant.

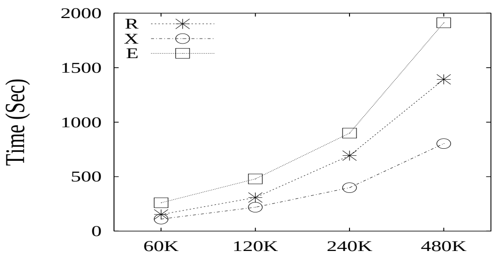


Figure 14: Scalability Test ($X_R = 4$ and $X_L = 16$)

6.3 Exp-3: Scalability Test

Figure 14 demonstrates the scalability of our algorithms by increasing the dataset sizes, for an XPath query `a//d` over the cross-cycle DTD (Fig. 11 (a)). We set $X_L = 16$, because the default $X_L = 12$ is not large enough for the XML generator to generate such large datasets. When the parameters are fixed, the XML generator can generate different sizes of XML databases but with the similar distributions in terms of heights/widths. The XML dataset size increases to 480,000 elements from 60,000 elements. We find that $\mathsf{CycleE_X}$ outperforms both SQLGen-R and CycleE noticeably, and SQLGen-R outperforms CycleE. When the dataset size is 480,000, the costs of CycleE and SQLGen-R are 2.4 times and 1.7 times of the cost of $\mathsf{CycleE_X}$, respectively. This shows that when dataset is large, our optimization technique is effective enough to outperform *with...recursive*, because it can reduce the number of LFP operators and unnecessary joins and unions.

6.4 Exp-4: Complex Cycles (Extracted from Real-Life DTDs)

We next show the results of testing XPath queries on the extracted 4-cycle BIOML DTD.

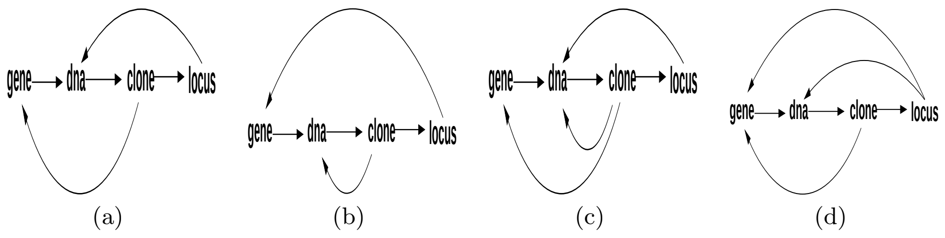


Figure 15: Different DTD graphs extracted from BIOML

Case	Query	n-Cycles	DTD Graph
2a	gene//locus	2	Fig. 15 (a)
2b	gene//locus	2	Fig. 15 (b)
2c	gene//dna	2	Fig. 15 (b)
3a	gene//locus	3	Fig. 15 (c)
3b	gene//locus	3	Fig. 15 (d)
4a	gene//locus	4	Fig. 11 (b)
4b	gene//dna	4	Fig. 11 (b)

Table 4: XPath queries over different DTD graphs extracted from BIOML

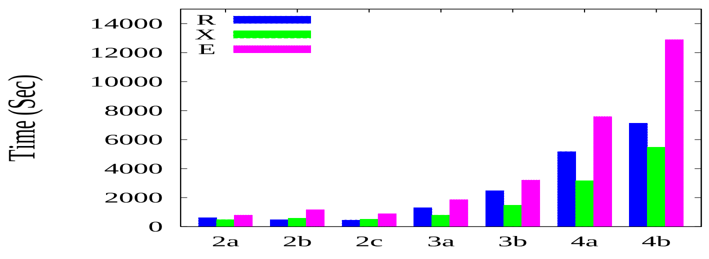


Figure 16: XPath queries on the extracted BIOML DTDs

We tested XPath queries over the extracted DTD graphs from BIOML. We considered four subgraphs of the BIOML DTD of Fig. 11 (b) in order to demonstrate the impact of different DTDs on the translated SQL queries. These subgraphs are shown in Fig. 15. The XPath queries tested on these extracted DTD graphs are summarized in Table 4.

All these XPath queries were run on the same dataset which was generated using the largest 4-cycle DTD graph extracted from BIOML (Fig. 11 (b)) with $X_R = 6$ and $X_L = 16$. Unlike Exp-1, we did not trim the XML trees generated by the IBM XML Generator. The generated dataset consists of 1,990,858 elements, which is 16 times larger than the dataset (120,000 elements) used in Exp-1. The sizes of relations for *gene*, *dna*, *clone* and *locus* are 354,289, 703,249, 697,060 and 236,260, respectively.

We show the query processing results in Fig. 16. We find that $\mathsf{CycleE_X}$ outperforms SQLGen-R and CycleE in all the cases, except case 2b. In case 4a, for example, SQLGen-R needs to use 7 joins and 7 unions in each iteration; CycleE needs to process 6 join, 2 LFP and 3 union operators; and $\mathsf{CycleE_X}$ uses 5 join, 1 LFP and 4 union operators. $\mathsf{CycleE_X}$ significantly outperforms SQLGen-R and CycleE because less number of join and LFP are used, while it uses more union operators than others. The cost of union is comparatively small, if one relation involved in the union operator is indexed.

DTD	n	m	c	CycleE		CycleE$_X$	
				`LFP`	`ALL`	`LFP`	`ALL`
Cross (Fig. 11(a))	4	5	2	5/9/6	38/78/51	2/2/2	7/11/8
BIOMLa (Fig. 15(a))	4	5	2	8/14/12	80/124/104	3/5/4	12/22/16
BIOMLb (Fig. 15(b))	4	6	3	6/14/11	50/94/75	2/5/3	9/20/14
BIOMLc (Fig. 15(c))	4	6	3	8/14/12	80/124/104	3/5/4	12/22/16
BIOMLd (Fig. 15(d))	4	7	4	8/14/12	88/134/112	3/5/4	13/23/17
GedML (Fig. 11(c))	5	11	9	6/22/16	154/222/188	2/8/4	12/27/19

Table 5: Number of Operations (min/max/average)

6.5 Exp-5: Number of Operations

We show the numbers of operations both in the resulting extended XPath expressions obtained from CycleE$_X$ and CycleE and in the resulting relational algebra (RA) in Table 5. Empirically, the lengths of the resulting extended XPath expressions and SQL are polynomial, even though in theory, the sizes of resulting extended XPath expressions are exponential, in the worst case, in terms of the size of $|G_D|$, based on [13].

In Table 5, the first column lists six DTD used in the testing. The second, third, and fourth columns indicate the numbers of nodes (n), edges (m), and simple cycles (c), respectively, in the DTD graphs. For each DTD, we enumerate all possible pairs of two nodes in the DTD, and select one as a start node (A) and the other as an end node (B). For each pair of A and B, we use CycleE and CycleE$_X$ to compute the extended XPath expression representing all paths from A to B, and then determine the number of operations in the resulting relational algebra (RA). They are shown in two groups in Table 5. The `LFP` and `ALL` show the numbers of LFP's and all operations used in extended XPath expressions in the format of (min/max/average). CycleE$_X$ outperforms CycleE in terms of the numbers of LFP and all operations used in all the cases.

7 Related Work

This is an extension of the earlier work [16] by including (a) the notion of extended XPath (Section 2) and its application in query translation and query answering (Section 3), (b) revised translation algorithms (Sections 4 and 5), in particular a new algorithm for handling the descendant axis of XPath; and (c) an extensive experimental study.

There has been a host of work on querying XML using an RDBMS, over XML data stored in an RDBMS or XML views published from relations (e.g., [11, 12, 33, 18, 27, 24, 42, 41]; see [28] for an excellent survey). However, as observed by [28], with the exception of [27], no algorithm has been published for handling recursive XML queries over recursive DTDs for schema-based XML storage or in the context of XML publishing. Closest to our work is [27], which proposed the first technique to rewrite recursive path queries over recursive DTDs to SQL with the SQL'99 recursion operator. We have remarked the differences between [27] and this work in Sections 1 and 3.

At least three approaches have been proposed to querying XML data stored in relations via DTD-based shredding. One approach is based on middleware and XML views, e.g., XPERANTO [42, 41] and SilkRoute [18]. In a nutshell, it provides clients with an XML view of the relations representing the XML data; upon receiving an XML query against the view, it composes the query with the view, rewrites the composed query to a query in a (rich) intermediate language supported by middleware, and answers the query by using the computing power of both the middleware and the underlying RDBMS. However, this approach is tempered by the following. First, it is nontrivial to define a (recursive) XML view of the relational data without loss of the original information. Second, it requires middleware support and incurs communication overhead between the middleware and the RDBMS. Third, as observed by [27], no algorithms have been developed for handling recursive queries over XML views with a recursive DTD.

Another approach is by providing an algorithm for rewriting XML queries into SQL (extended with a recursion operator), which is the approach adopted by this work. To this end, translation and optimization techniques have been proposed for translating XSLT queries [24], XQuery [11, 12, 29, 33] and (recursive) path queries [27]. While the algorithms of [24, 11, 12, 29, 33] cannot handle query translation in the presence of recursive DTDs, their optimization techniques by leveraging, e.g., integrity constraints [12, 29], virtual generic schema and query normalization [33], dynamic interval encoding [11] and aggregation handling [24] are complementary to our work. Some of these, along with techniques for query pruning and rewriting [17], minimizing the use of joins [30], multi-query [39] and recursive-query optimization [40], can be incorporated into our translation framework.

The third approach is based on indexing structures, which associate each node with, e.g., the pair of preorder traversal and *range* numbers, where the latter is the maximum number of descendants of the node (e.g., [32, 44]). One can preserve the indexes when storing XML data in relations, and translate certain recursive XPath queries to SQL without using any recursion operator by capitalizing on the indexes. This approach is hampered by the following problems. First, the indexes introduce additional overhead when storing and querying the data. Worse still, the cost of the maintenance of the indexes may become prohibitive expensive when the data is frequently changed. In contrast, our approach does not incur extra cost in the dynamic context. Second, it does not help query answering for XML views, as opposed to this work.

There has also been recent work on query answering for virtual XML views in the native XML setting [15, 14]. This issue was studied in [14] for nonrecursive XML views, and it was revisited for recursive XML views in [15]. As remarked earlier, it was shown in [15] that for recursive XML views, query rewriting is not closed for XPath, but it is closed for regular XPath; however, the rewriting incurs an exponential-time lower bound even for nonrecursive XML views. To avoid the exponential blowup, [15] proposed a notion of automata to represent the rewritten regular XPath queries, and developed algorithms for evaluating these automata on XML data. Unfortunately, those automata cannot be directly translated into SQL with LFP. In contrast, this work introduces extended XPath and shows that extended XPath expressions can be translated into equivalent SQL queries. Regular XPath was introduced in [34]. Extended XPath proposed by this work is an extension of regular XPath by allowing bindings of variables and sub-queries.

Surveys on recursive and cyclic query processing strategies include [5, 25]. For OODBs, [26] introduced techniques for processing cyclic queries restricted to 1-cycle queries. [9] proposed optimization techniques for generalized path expressions based on OO algebraic transformation rules. These techniques are not directly applicable to query translations from XML to SQL.

8 Conclusion

We have proposed a new approach to translating a practical class of XPath queries over (possibly recursive) DTDs to SQL queries with a simple LFP operator found in many commercial RDBMS. The novelty of the approach consists in (1) a notion of extended XPath expressions capable of capturing DTD recursion and XPath recursion in a uniform framework; (2) an efficient algorithm for translating an XPath query over a recursive DTD to an equivalent extended XPath expression that characterizes all matching paths, without incurring exponential blowup; and (3) an efficient algorithm for rewriting an extended XPath expression into an equivalent sequence of SQL queries; These provide not only the capability of answering important XPath queries within the immediate reach of most commercial RDBMS, but also the query answering ability for certain XML views.

Several extensions are targeted for future work. We recognize that several factors affect the efficiency of the SQL queries produced by our translation algorithms, and we are currently developing a cost model in order to provide better guidance for XPath query rewriting. We are also exploring techniques for multi-query and recursive-query optimization [39, 40] to simplify the SQL queries produced. Finally, we plan to extend our algorithms to handle more complex XML queries, over XML data stored in an RDBMS or (virtual) XML views of relational data.

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